THE EFFECT OF OPTICAL FIBER NON-LINEARITIES ON INTENSITY MODULATED MULTIPLE OPTICAL CARRIER SYSTEMS

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CERTIFICATE

It is certified that the work contained in the th
entitled "THE EFFECT OF OPTICAL FIBER NON-LINEARITIES ON INTEN
MODULATED MULTIPLE CARRIER SYSTEMS" by Rajesh M.K. has
carried out under my supervision and that this work has not
submitted elsewhere for a degree.

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TO MY

PARENTS

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I take this opportunity to express my indebteness for my friends, and my Aunt and Uncle who have made my stay at I.I.T. Kanpur homely and pleasurable.

RAJESH M. K.

CONTENTS

ABSTRACT

The contributions of the self-phase modulation and cross phase modulation, the two optical fiber non-linearities, to the interference power spectrum for an intensity modulated sub-carrier multiplexed, multiple optical carrier systems have been analysed. The equations for interference power spectrum has been derived for a two optical carrier case and then generalized to N-optical carriers under some specific cases and assumptions. The signal to interference ratios have been calculated and plotted against the laser spectral width. The degradation of the above system due to increase in the number of optical carriers is also shown.

LIST OF FIGURES

CHAPTER - 1

INTRODUCTION

Optical fiber has many excellent features as a transmission medium. Among those a wide bandwidth availability is the most promising. Since the optical carrier frequency is in the range of 10^{14} Hz even 0.1 % utilization of the bandwidth is sufficient to include ten thousand TV channels of 5 MHz bandwidth with a spacing of 5 MHz between them. The enormous bandwidth capacity of the fiber optic link is tailor made for multiple access light wave systems, which include local area networks and metropolitan networks.

1.1 SCHEMES IN MULTIPLE ACCESS SYSTEMS

The two types of schemes which are used in multiple access systems are :

a) Time division multiplexing (TDMA)

b) Frequency division multiplexing (FDMA)

Multiple access systems differ from wide-band links in that each user generally requires only a small fraction of the total data throughput. While using the TDMA, each receiver must receive all the transmitted bits and select the appropriate bits for its use. This unnecessarily calls for receivers with greater sensitivity than that is required.

The FDMA, an alternative scheme for bandwidth utilization, can be classified into two types :

i) Optical frequency division multiplexing (OFDM)

ii) Sub-carrier multiplexing (SCM)

In the OFDM, the channel selection and other functions are carried out at optical frequency. This requires that the laser line width be very small and operated only in single mode conditions. At present this method of implementation is not cost effective.

In SCM system, as shown in Fig. 1.1, data from each channel are used to modulate microwave subcarriers, f_1, $f_2, \ldots, f_n$, which are then used to intensity modulate a single optical carrier. Fig. 1.2 shows optical carriers being modulated by a different set of subcarriers, and the signals from each set being combined in a power combiner and then sent in a single fiber.

1.2 AN OVERVIEW OF OPTICAL FIBER NON-LINEARITIES IN SINGLE MODE FIBER

In its simplest form, an optical fiber consists of a central core surrounded by a cladding layer whose refractive index is slightly lower than that of core (step index fiber). The two parameters which characterize the fiber are the normalized difference of refractive indices of the core (n_1) and cladding (n_2), [1]

$$\Delta = \frac{n_1 - n_2}{n_1} \tag{1.1}$$

and the normalized frequency V defined by

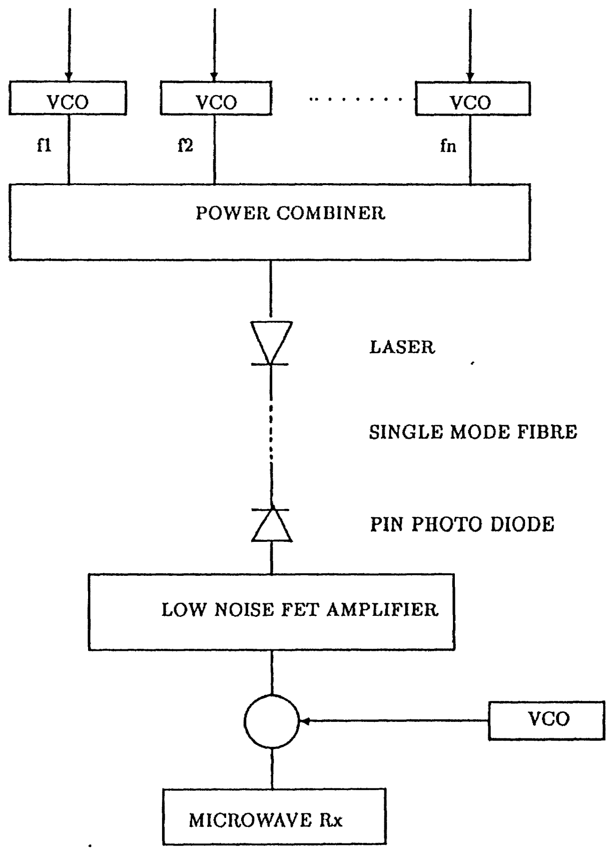


Figure 1.1: An SCM Light Wave System

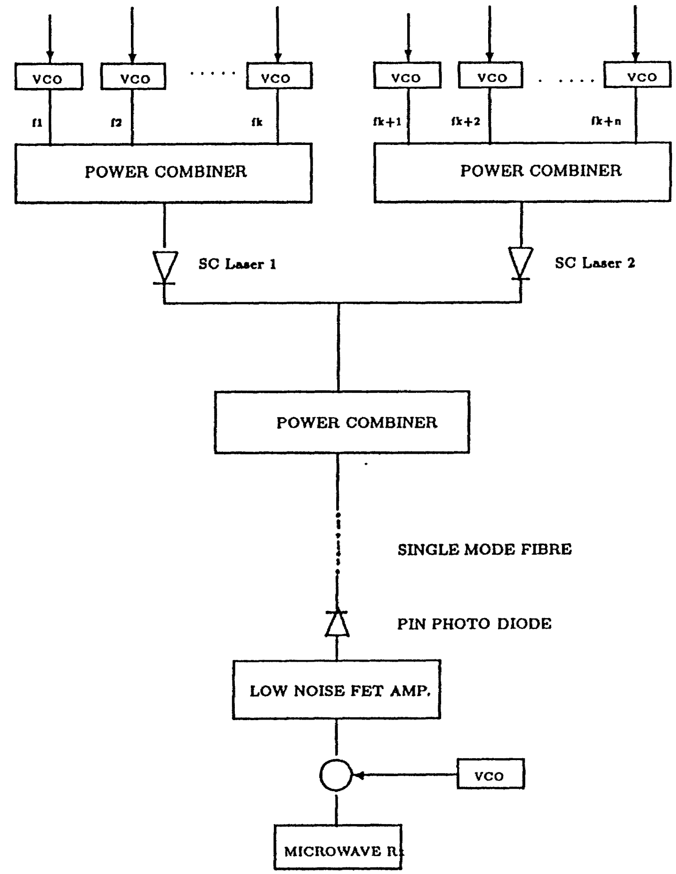


Figure 1.2: A Two Stage SCM Light Wave System

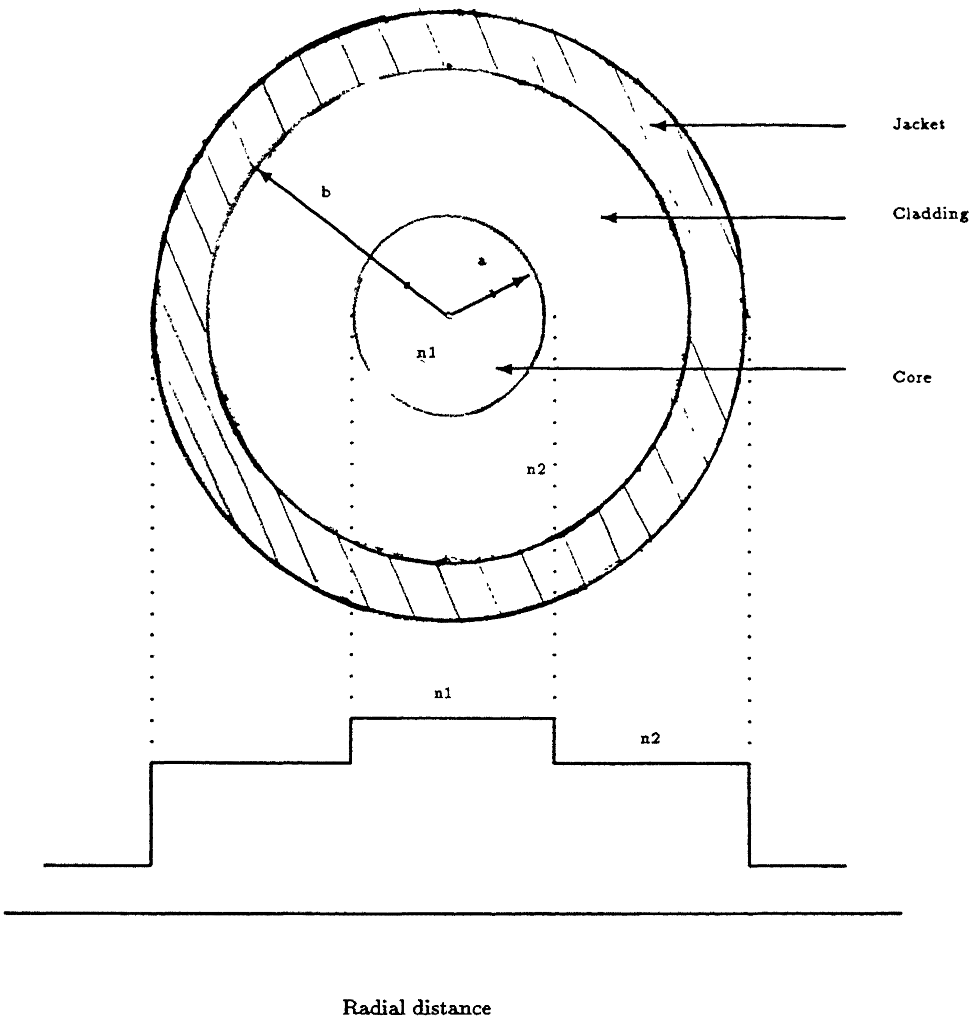


Figure 1.3: A Cross Section Of A Step Index Fibre

$$V = K_o a \left(n_1^2 - n_2^2\right)^{1/2} \qquad (1.2)$$

where $K_o = 2\pi/\lambda$

a is the core diameter

and λ is the wavelength of the transmitted wave.

It is shown that if $V < 2.405$, the fiber supports only a single mode. The typical values of the core diameter are 25-30 μm for a multimode and 2-4 μm for a single mode fiber.

1.2.1 Attenuation

An important design parameter is the power loss during transmission of optical signals inside the fiber. If P_o is the power launched at the input of the fiber of length L, the transmitted power is given by

$$P_T = P_o \exp(-\alpha L) \qquad (1.3)$$

α = attenuation constant expressed nepers/km.

Material absorption and Rayleigh scattering are the major factors contributing to fiber loss. The fiber loss at present is very low, about 0.12-0.15 dB/km at $\lambda = 1.55$ μm.

1.2.2 Dispersion

In a single mode fiber, the only dispersion which is of importance is the group velocity dispersion. This is caused when

different frequencies travel with different velocities in the fiber. The propogation constant β is given by

$$\beta = \frac{n\omega}{c} \tag{1.4}$$

The refractive index is not a constant but a function of frequency. This relation is given by Sellemier's equation [1]

$$n(\omega) = 1 + \sum_{j=1}^{m} \frac{B_j\ \omega_j^2}{\omega_j^2 - \omega^2} \tag{1.5}$$

ω_j is the resonance frequency at which medium absorbs the optical radiation

B_j is the strength of resonance.

The group velocity is the velocity with which the envelope of the optical field moves in the fiber and is defined by

$$v_g = \left[1/(d\beta/d\omega)\right] \tag{1.6}$$

Substituting eqs. (1.4) and (1.5) in eq. (1.6), we see that v_g is dependent on frequency. This type of dependence causes chromatic or material dispersion. A measure of variation of group-velocity with frequencies is given by the parameter β_2, i.e., group velocity dispersion parameter [1],

$$\beta_2 = \frac{d\beta_1}{d\omega} = \frac{d^2\beta}{d\omega^2} \tag{1.7}$$

where $\beta_1 = \frac{d\beta}{d\omega}$

1.2.3 Self Phase Modulation (SPM) and Cross Phase Modulation (XPM)

As seen from the above discussion the variation of refractive index with frequency gives rise to chromatic dispersion. Another kind of non-linearity is the variation of the refractive index due to the incident intensity of the light. This effect is known as the self phase modulation (SPM), and now the refractive index of the fiber becomes

$$\bar{n}\ (\omega,\ E(t)) \ = \ n(\omega) + n_2 |E(t)|^2 , \tag{1.8}$$

where $|E(t)|^2$ is the instantaneous optical power,

$E(t)$ is electric field in volts/metre,

$n(\omega)$ is the linear part of refractive index and n_2 is the non-linear refractive index coefficient.

The intensity dependence of the refractive index leads to a large number of interesting non-linear effects. As evident from eq. (1.8), the instantaneous phase is given by

$$\phi(t) \ = \left[n(\omega) + n_2 |E(t)|^2 \right] \ \frac{2\pi}{\lambda}\ L \tag{1.9}$$

L = fiber length

λ = wavelength

n_2 = non-linear refractive index coefficient

$n(\omega)$ = refractive index

The non-linear phase shift due to SPM is

$$\phi_{nL} \ = \ n_2\ L\ \frac{2\pi}{\lambda}\ (|E(t)|^2) \tag{1.10}$$

The cross phase modulation (XPM) refers to the effects of the intensity of two copropagating fields say $E_1(t)$ and $E_2(t)$ on the phase of the other. If $\tilde{E}_1(t)$ and $\tilde{E}_2(t)$ are the envelopes of the two copropagating optical fields then the non-linear phase shift for the first field is given by

$$\phi_{NL}(t) = n_2 K_o L(|E_1(t)|^2 + 2|\tilde{E}_2(t)|^2) \qquad (1.11)$$

and for the second field is given by

$$\phi_{NL}(t) = n_2 K_1 L(|\tilde{E}_2(t)|^2 + 2|\tilde{E}_1(t)|^2) \qquad (1.12)$$

where $K_o = 2n/\lambda_1$, λ_1 is the wavelength of first wave,
$K_1 = 2n/\lambda_2$, λ_2 is the wavelength of second wave.

It can be that the XPM factor for two equally intense electric fields is twice as compared with that of the SPM factor.

1.3 DETECTION SCHEME

We can broadly classify the detection schemes used in lightwave communication into two types.

a) Direct Detection (DD)

b) Coherent Detection (CD)

It is shown that the signal to noise ratio for DD system is lower at least by a factor of eight as compared to that of CD system. This advantage, however, is at the cost of receiver complexity and increase in the cost of a CD receiver. In a sub-carrier multiplexed intensity modulation system with direct

detection, the components required for detection are a photo-diode followed by a low noise amplifier and a microwave receiver. The microwave receiver components are easily available and standardized. The source line width in a DD system is not as stringent as in the case of CD systems. In the present work the intensity modulated system with direct detection is considered.

1.4 MODULATION SCHEME

Modulation schemes can be broadly classified into :

1) Intensity Modulation (IM) (analog/digital)
2) Coherent Modulation (CM) (analog/digital)
   a) Frequency Modulation (FM) (analog/digital)
   b) Phase Modulation (PM) (analog/digital)

Intensity modulation is a simple modulation scheme to implement. By varying the laser bias current about a operating point the output power of the laser can be modulated accordingly. This can be detected by a direct detection scheme.

Phase modulation and frequency modulation require that the laser spectral width be very narrow and the photo detector at the receiver has a flat response. These factors increase the cost of the system when we are considering multiple access system with coherent modulation schemes.

1.5 OBJECTIVES AND SCOPE OF PRESENT WORK

The effect of SPM and XPM on a single optical carier IM system is of little consequence, due to the inherent nature of square law detection, where the envelope can be recovered easily.

Consider the IM wave with m(t) being the message and frequency ω_1 in optical domain

$$x(t) = (1+m(t))^{1/2} \cos(\omega_1 t) \tag{1.13}$$

Due to the SPM, the signal would become

$$x(t) = (1+m(t))^{1/2} \cos(\omega_1 t + K(1+m(t))) \tag{1.14}$$

K being SPM factor. During the detection process the signal is squared.

$$(x(t))^2 = (1+m(t))\left[1/2 + 1/2 \cos(2\omega_1 + 2K(1+m(t)))\right] \tag{1.15}$$

$$= \frac{(1+m(t))}{2} + \frac{(1+m(t))}{2}\left[\cos(2\omega_1 + 2K(1+m(t)))\right] \tag{1.16}$$

The second term is filtered out and the message process m(t) is easily recovered. Now consider a two optical carrier system. The individual signals are combined and sent in the optical fiber. The composite signal is given by

$$x(t) = \left[(1+m(t))^{1/2} \cos(\omega_1 f)\right] + \left[(1+m_2(t))^{1/2} \cos(\omega_2 f)\right] \tag{1.17}$$

It can be shown that after being passed through the photo-detector the signal has 3 components; they are

$$(1+m_1(t)),\ (1+m_2(t)),\ \text{and}$$

$$(1+m_1(t))^{1/2}\,(1+m_2(t))^{1/2}\cos((\omega_1-\omega_2)t+\phi(t))\ \text{respectively} \qquad (1.18)$$

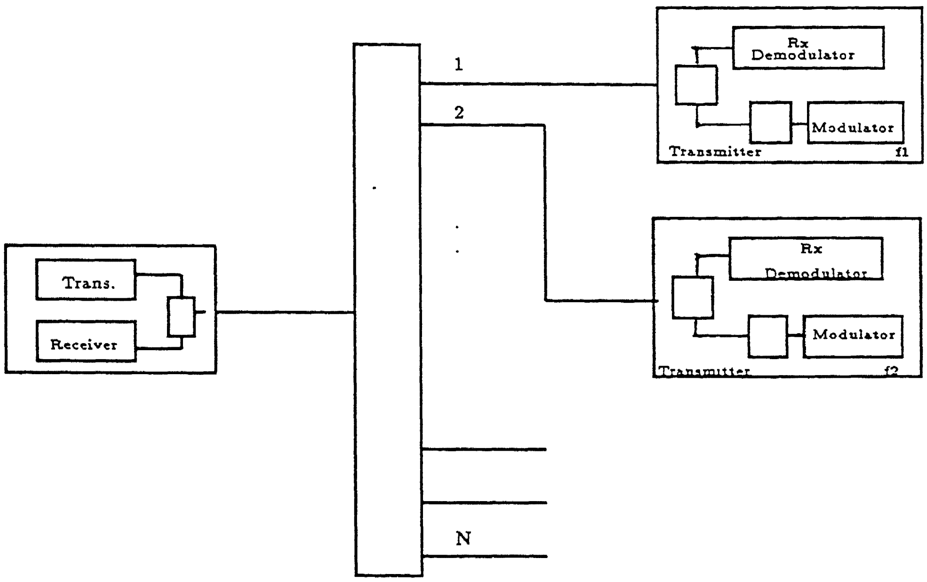
Not only the message processes $m_1(t)$ and $m_2(t)$ are recovered but also the cross term is centered at $|\omega_1-\omega_2|$. Without SPM and XPM taking into consideration, the phase factor $\phi(t)$ in the cross term goes to zero. It becomes evident that the SPM and XPM tend to spread out the power spectral density of the cross term giving rise to more interference.

The purpose of this work is to analyse the amount of interference caused in a multiple optical carrier, sub-carrier multiplexed intensity modulated system, due to self-phase modulation and cross-phase modulation, the two non-linearities of the fiber.

1.6 PRACTICAL APPLICATION

Consider a scheme where N customers are received by a local exchange, as shown Fig. 1.4 [2].

The exchange provides customers' services such as voice, data, and video on different sub-carrier frequencies. When the customers want to communicate with the exchange, each customer transmits the required message by modulating at a different microwave sub-carrier. The detector then detects the sum of all the optical signals. This mixing process gives rise to interference noise, and ultimately can limit the number of optical carriers and the transmission bandwidth of the message source.



1:N COUPLER

Figure 1.4: A Bidirectional SCM Lightwave System

The limitation posed by such a mixing process is analysed in this thesis.

1.7 ORGANIZATION OF THESIS

Chapter 2 discusses about the SPM and XPM factors, the laser phase noise, and problem formulation taking into effect SPM and XPM factors and laser phase noise.

Chapter 3 deals with derivation of the cross power spectral density for specific cases. The signal to interference ratio (SIR), is calculated for various line widths of the laser source. The variation of SIR with number of laser sources used is estimated.

Chapter 4 discusses the inference and scope for future work.

CHAPTER - 2

PROBLEM FORMULATION

2.1 CALCULATION OF SPM AND XPM FACTORS

As seen earlier the non-linear phase shift depends on the intensity of propagating radiation, vide eq. (1.10). If we consider N optical carriers travelling in the fiber with intensity modulated envelopes of $\tilde{E}_1(t)$, $\tilde{E}_2(t)$ $\tilde{E}_3(t)$ ... $\tilde{E}_N(t)$ then the phase change in channel j is given by

$$\phi_j = \nu \, L(eff) \left[|\tilde{E}_j(t)|^2 + 2 \sum_{\substack{m=1 \\ m \neq j}}^{N} |\tilde{E}_m(t)|^2 \right] \tag{2.1}$$

ν is called the non-linearity parameter [1]

and $$\nu = \frac{n_2 \omega}{c \, A_{eff}} \tag{2.2}$$

ω = frequency of propagating lightwave in radians/sec

A_{eff} = effective area in fiber from which the maximum power is transmitted

A_{eff} is of the order of 10-20 μm^2 in visible region and 50-80 μm^2 in 1.5 μm wavelength region. n_2 the non-linear refractive index coefficient is

$$n_2 = 3.2 \times 10^{-16} \ cm^2/W$$

$$L_{eff} = \frac{1}{\alpha} \exp(-\alpha L) \tag{2.3}$$

α = attenuation constant $\cong 0.046$ km^{-1} (0.2 dB/km). If total length L of the fiber is large, then

$$L_{eff} \cong 1/\alpha \tag{2.4}$$

ν varies between 2-3 W^{-1} km^{-1} as the frequencies vary between visible and infra-red region.

We define a self-phase modulation factor K

as

$$K = \nu L_{eff} \tag{2.5}$$

If we assume $L_{eff} \cong 1/\alpha = 1/0.046$ km and ν varies between 2-30 W^{-1} km^{-1} then K varies between 43.47-652.17 W^{-1}. The cross phase modulation factor is twice that of SPM.

2.2 ANALYSIS OF LASER PHASE NOISE

The power spectrum of the laser source is not delta function but is considered as a lorentzian shaped power spectrum.

The ideal laser source is represented as

$$x(t) = A \exp(j\omega_o t), \tag{2.6}$$

$$\mathcal{F}_T(x(t)) = A\,\delta(\omega - \omega_o), \tag{2.7}$$

where $\mathcal{F}_T$ [ ] represents the Fourier transform operator.

To incorporate the spread in the power spectrum, the laser source is represented as

$$x(t) = A \exp\left[j(\omega_o t + \phi(t)\right] \tag{2.8}$$

where $\phi(t)$ is usually modelled as a continuous path Brownian motion process with zero mean. The characteristic of Brownian motion stochastic process is that, the incremental processes $|\phi(t) - \phi(t + \tau)|$ are independent random variables with zero mean and with gaussian distribution [3].

From the Wiener-Khintchine theorem we know that the Fourier transform of the autocorrelation function yields the power spectral density (PSD) [ ],

$$\text{PSD} = \mathcal{F}_T \left\{ \mathcal{E} \left\{ x^*(t)\ x(t + \tau) \right\} \right\} \tag{2.9}$$

$$= \mathcal{F}_T \left\{ \mathcal{E} \left\{ A \exp\left[j(\omega_o \tau - \phi(t) + \phi(t + \tau)) \right] \right\} \right\} \tag{2.10}$$

$$= \mathcal{F}_T \left\{ A \exp(j\omega_o \tau)\ \mathcal{E} \left\{ \exp(j(\phi(t+\tau) - \phi(t)) \right\} \right\} \tag{2.11}$$

$$= \mathcal{F}_T \left\{ A \exp(j\omega_o \tau) \exp(-\pi|\tau|/\tau_c) \right\} \tag{2.12}$$

where τ_c is called the coherence time, and $\mathcal{E}$ denotes the expectation operator. For the derivation of eq. (2.12), refer to Appendix A.1.

Carrying out the above Fourier transform

$$\text{PSD} = \frac{A.2\ (\pi\Delta v)^{-1}}{1 + \left[\frac{2\pi(f - f_o)}{\pi\Delta v} \right]^2} \tag{2.13}$$

where $\Delta v = 1/\tau_c$ is called the half power bandwidth of the laser source. The derivation of the above equation is given in Appendix A.2.

The equation (2.13) denotes a Lorentzian shaped spectra.

Another important result which we will be making use of is

$$\mathcal{E}\left\{ \exp\,(j(\phi(t) + \phi(t+\tau))\right\} = 0 \qquad (2.14)$$

we can write the above equation as

$$\mathcal{E}\left\{ \exp\,(j(\phi(t) - \phi(t+\tau)) + j2\phi(t+\tau))\right\} = 0 \qquad (2.15)$$

Since the characteristic of Brownian process is that, the processes $|\phi(t) - \phi(t+\tau)|$ are independent, we can write eq. (2.15) as

$$\mathcal{E}\left\{ \exp\,(j(\phi(t) - \phi(t+\tau))\right\} \mathcal{E}\left\{\exp\,(j2\phi(t+\tau))\right\} = 0 \qquad (2.16)$$

It is usually assumed that the underlying process is a gaussian zero mean non-stationary random process [4]. The variance of the gaussian random process increases linearly with time. In such a situation the integral for t >> 0 goes to zero.

i.e. $$\mathcal{E}\left\{ \exp\,(j2(\phi(t+\tau))\right\} = 0\,, \qquad (2.17)$$

which is derived in Appendix A.3.

2.3 SPECTRAL ANALYSIS - MIXING OF OPTICAL FIELDS

2.3.1 Mathematical Preliminaries

Let the real signal

$$y(t) = A(t) \cos (2\pi \nu t + \phi(t)) \tag{2.18}$$

be a random stationary narrow-band optical process.

Let $\tilde{y}_c(t) = A(t) \exp (j\phi(t))$ be the complex envelope of the complex signal. The subscript c denotes a complex signal.

$$y_c(t) = \tilde{y}_c(t) \exp(j2\pi\nu t) = A(t) \exp(j\phi(t)) \exp(j2\pi\nu t), \tag{2.19}$$

which is centered around the optical carrier frequency ν and is tightly band limited around the optical carrier frequency. The real signal y(t) is

$$y(t) = \mathrm{Re}\left\{y_c(t)\right\} \tag{2.20}$$

By the Weiner-Khintchine theorem the power spectrum of Stochastic process is the Fourier transform of its autocorelation.

The autocorrelation of the complex signal is

$$\Gamma_c(\tau) = \mathcal{E}\left\{\tilde{y}_c(t+\tau)\ \tilde{y}_c^{*}(t))\right\} \exp(j2\pi\nu\tau) = \tilde{\Gamma}_c(\tau) \exp (j2\pi\nu\tau), \tag{2.21}$$

where $\tilde{\Gamma}_c(\tau)$ is the complex envelope of the autocorrelation.

The power spectrum of the complex signal is $S_c(t)$ and it is given by

$$S_c(t) = \mathcal{F}_T \left\{\Gamma_c(\tau)\right\} \tag{2.22}$$

To find the power spectrum of the real signal we have to find its autocorrelation function and then take the Fourier transform of the autocorrelation function,

$$\Gamma(t,\ t+\tau) = \mathcal{E}\left[\mathrm{Re}\left\{y_c(t+\tau)\right\}\ \mathrm{Re}\left\{y_c(t)\right\}\right] \tag{2.23}$$

Since, it is known that,

$$\mathrm{Re}\ \{\ A_c\ \}\mathrm{Re}\ \{\ B_c\} = 1/2\ \mathrm{Re}\ \{\ AB_c\} + 1/2\ \mathrm{Re}\ \{\ AB_c^*\ \} \tag{2.24}$$

we have

$$\Gamma(t,\ t+\tau) = 1/2\ \mathrm{Re}\ \mathcal{E}\left\{y_c(t+\tau)\ y_c^*(t)\right\} + 1/2\ \mathrm{Re}\ \mathcal{E}\ \left\{y_c(t+\tau)\ y_c(t)\ \right\} \tag{2.25}$$

$$\Gamma(t,\ t+\tau) = 1/2\ \mathrm{Re}\ (\Gamma_c(\tau)) + 1/2\ \mathrm{Re}\mathcal{E}\left[\langle\tilde{y}_c(t+\tau)\tilde{\ }y_c(t)\ \exp\ (j2\pi\nu(t+\tau)\rangle\right] \tag{2.26}$$

where <.> indicates time average.

Since we are interested in the average power spectral density we have to do a combination of statistical and time averaging. It is shown in [4] that the second term of eq. (2.26) vanishes. Therefore, the autocorrelation of the real signal is half the real part of the complex signal.

$$\Gamma_c(\tau) = 1/2\ \mathrm{Re}\ \{\ T_c(\tau)\ \} \tag{2.27}$$

2.3.2 Problem Definition

The general equation for the electric field of an amplitude modulated signal is

$$E_{1c}(t) = (p_1(t))^{1/2} \exp(j\omega_1 t) \tag{2.28}$$

where $(p_1(t))^{1/2}$ is the modulated envelope and ω_1 is an optical carrier frequency. If there are two optical carriers ω_1 the ω_2, the combined equation of the electric field is

$$E_c(t) = (p_1(t))^{1/2} \exp(j\omega_1 t) + (p_2(t))^{1/2} \exp(j\omega_2 t) \tag{2.29}$$

where $(p_2(t))^{1/2}$ is the modulated envelope and ω_2 is the second optical carrier frequency.

If the fiber optic link does not introduce any distortion, then the signal incident on the photo detector would be

$$[\ E(t)\ ]^2 = [\ \mathrm{Re}\ \{\ E_{1c}(t)\ \} + \mathrm{Re}\ \{\ E_{2c}(t)]^2 \tag{2.30}$$

At the output of the photo detector we would have

$$i(t) = \chi\ (E(t))^2\ (*)\ h_d(t)\ , \tag{2.31}$$

where χ is the responsivity of photo detector, amperes/watts

$i(t)$ is the output current

$h_d(t)$ is the impulse response of the photo detector

(*) denotes the convolution operation.

For the purpose of analysis, we assume that the photo detector be an ideal low pass filter with a cut off frequency which is much lower than the optical frequency. The resulting photo current after low pass filtering of eq. (2.30) yields [5]

$$i(t) = p_1(t) + p_2(t) + 2\,\mathrm{Re}\,\{ (p_1(t)\, p_2(t))^{1/2} \exp(j(\omega_1 - \omega_2)t) \}$$

$$= i_1(t) + i_2(t) + i_x(t) \qquad (2.32)$$

The first two terms $i_1(t)$ and $i_2(t)$ give us the modulated envelope. The third term $i_x(t)$ however, is the result of mixing process and represents an undesired interference component. If its power spectrum lies within the bandwidth of the power spectrum of the envelopes $i_1(t)$ and $i_2(t)$.

To find the power spectral density of the resulting photo current $i(t)$, we must consider the Fourier transform of the autocorrelation, i.e.

$$\mathcal{F}_T \left[\mathfrak{X} \left[i(t+\tau)\; i^*(t) \right] \right] = S(f) \qquad (2.33)$$

With the assumption that $E_{1c}(t)$ and $E_{2c}(t)$ are zero mean independent processes, it is shown that $S(f)$ can be written as [5]

$$S(f) = S_1(f) + S_2(f) + S_{12}(f) + \mathcal{F}_T \left\{ \overline{\Gamma_{E1}\; \Gamma_{E2}} \right\} \qquad (2.34)$$

where,

$S_1(f)$ is the power spectrum of $p_1(t)$

$S_2(f)$ is the power spectrum of $p_2(t)$

$S_{12}(f)$ is $\mathcal{F}_T \{ \mathfrak{X}(i_1(t)) * \mathfrak{X}(i_2(t)) \}$, where

$i_1(t)$ and $i_2(t)$ contain sinusoidal modulation terms. The Fourier transform shown above yields the impulsive cross power spectrum,

$$S_{12}(f) = I_1\, I_2\, \delta(f) ,$$

where I_1 and I_2 are two constants representing the average DC values of $i_1(t)$ and $i_2(t)$. Note that $\overline{\Gamma_{E1}\,\Gamma_{E2}}$ denotes the time averaged autocorrelation of the signals. Since $E_1(t)$ and $E_2(t)$ are independent, then $\overline{\Gamma_{E1}\Gamma_{E2}}$ separates into $\overline{\Gamma}_{E1}\,\overline{\Gamma}_{E2}$, that is they are time separable [5]. Therefore,

$$\overline{\Gamma_{E1}\,\Gamma_{E2}} = \overline{\Gamma}_{E1}\,\overline{\Gamma}_{E2} = \mathfrak{F}\left\{ \overline{E_1(t+\tau)\,E_1(t)} \right\} \mathfrak{F}\left\{ \overline{E_2(t+\tau)\,E_2(t)} \right\}$$

where
$$\begin{aligned} E_1(t) &= \mathrm{Re}\,\{ E_{1c}(t) \} \\ &= \mathrm{Re}\,\{ \tilde{E}_{1c}(t)\, \exp\,(j2\pi\, \nu_1 t) \} \end{aligned}$$

and
$$\begin{aligned} E_2(t) &= \mathrm{Re}\,\{ E_{2c}(t) \} \\ &= \mathrm{Re}\,\{ \tilde{E}_{2c}(t)\, \exp\,(j2\pi\, \nu_2 t) \} . \end{aligned}$$

Note that the over bar is used to denote time averaging in the above equations.

a) The Effect of Adding Laser Phase Noise

As shown previously, the laser phase noise would add a extra factor $\exp\,(j\phi_1(t))$ to the signals, i.e. consider $E_{1c}(t)$ and $E_{2c}(t)$ are two independent optical fields with a modulated envelope

$$E_{1c}(t) = (p_1(t))^{1/2} \exp j(\phi_1(t) \exp\,(j\omega_1 t) \qquad (2.35)$$

and

$$E_{2c}(t) = (p_2(t))^{1/2} \exp j(\phi_2(t) \exp\,(j\omega_2 t) \qquad (2.36)$$

Now the complex envelopes $\tilde{E}_{1c}(t)$ and $\tilde{E}_{2c}(t)$ are

$$\tilde{E}_{1c}(t) = (p_1(t))^{1/2} \exp(j\phi_1(t) \tag{2.37}$$

$$\tilde{E}_{2c}(t) = (p_2(t))^{1/2} \exp(j\phi_2(t) \tag{2.38}$$

After squaring of the combined fields at the photo detector the cross term can be represented as

$$i_x(t) = 2 \text{ Re} \left\{ \tilde{E}_{1c}(t) \, \tilde{E}_{2c}(t) \exp(j2\pi\nu_1 t) \times \exp(-j2\pi\nu_2 t) \right\} \tag{2.39}$$

Note from eq. (2.34) that the terms contributing to interference power spectrum is nothing but the Fourier transform of the time average autocorrelation of the cross term.

Let Γ_{E1} and Γ_{E2} represent the autocorrelation of $E_1(t)$ and $E_2(t)$, then

$$\Gamma_{E1}\,\Gamma_{E2} = 1/2 \text{ Re} \left\{ \mathfrak{X} \left[E_{1c}(t+\tau) \, E_{2c}^*(t+\tau) \, E_{1c}^*(t) \, E_{2c}(t) \right] \right\}$$

$$+ \; 1/2 \text{ Re} \left\{ \mathfrak{X} \left[E_{1c}(t+\tau) \, E_{2c}^*(t+\tau) \, E_{1c}(t) \, E_{2c}^*(t) \right] \right\} \tag{2.40}$$

To calculate the interference power spectrum, we should now calculate the average power spectrum of the first term, i.e.

$$\mathcal{F}_T \Big[1/2 \text{ Re} \Big[\mathfrak{X} \left[p_1(t+\tau) \, p_1(t) \, p_2(t) \, p_2(t+\tau) \right]^{1/2}$$

$$x \exp(j(\phi_1(t+\tau) - \phi_1(t)) - j(\phi_2(t+\tau) - \phi_2(t)) \exp(j2\pi\Delta\nu\tau) \Big] \Big] \tag{2.41}$$

where,

$\Delta\nu = \nu_1 - \nu_2$ is the difference frequency,

$\phi_1(t+\tau) - \phi_1(t)$ is the incremental phase noise process due to laser source I,

$\phi_2(t+\tau) - \phi_2(t)$ is the incremental phse noise process due to laser source II, and

$p_1(t)$ and $p_2(t)$ are the envelopes of the two signals.

(b) To Account for SPM and XPM Effectson Signal

Now the signal leaving the optical fiber has to be further refined to consider to effect of SPM and XPM. As before we consider complex signals $E_{1c}(t)$ and $E_{2c}(t)$. The incident intensity on the photo detector would be

$$[\ E(t)\ ]^2 = (\mathrm{Re}\ \{\ E_{1c}(t) + \mathrm{Re}\ \ E_{2c}(t)\ \}^2$$

where $E_{1c}(t)$ and $E_{2c}(t)$ are given by

$$E_{1c}(t) = (p_1(t))^{1/2} \exp(j\omega_1 t + jK_1 p_1(t) + j2K_1 p_2(t) + j\phi_1(t)) \tag{2.42}$$

$$E_{2c}(t) = (p_2(t))^{1/2} \exp(j\omega_2 t + jK_2 p_2(t) + j2K_2 p_1(t) + j\phi_2(t)) \tag{2.43}$$

$$\text{and,}\quad K_1 = \text{SPM factor 1} = \gamma_1 L_{eff} = \frac{n_2\ \omega_1\ L_{eff}}{C\ A_{eff}} \tag{2.44}$$

$$2K_1 = \text{XPM factor 1} \tag{2.45}$$

$$K_2 = \text{SPM factor 2} = \gamma_2 L_{eff} = \frac{n_2\ \omega_2\ L_{eff}}{C\ A_{eff}} \tag{2.46}$$

$$2K_2 = \text{XPM factor 2}$$

$p_1(t)$ and $p_2(t)$ contain the message processes which may represent any type of modulation scheme. $\phi_1(t)$ and $\phi_2(t)$ are laser phase noise of sources I and II respectively.

The above equation can be written conveniently as

$$E_{1c}(t) = (p_1(t))^{1/2} \exp(j\psi_1(t)) \exp(j\omega_1 t) = \tilde{E}_{1c}(t) \exp(j\omega_1 t) \tag{2.47}$$

$$E_{2c}(t) = (p_2(t))^{1/2} \exp(j\psi_2(t)) \exp(j\omega_2 t) = \tilde{E}_{2c}(t) \exp(j\omega_2 t) \tag{2.48}$$

where

$$\psi_1(t) = K_1 p_1(t) + 2K_1 p_2(t) + \phi_1(t)$$

$$\psi_2(t) = K_2 p_2(t) + 2K_1 p_1(t) + \phi_2(t)$$

Since $p_1(t)$ and $p_2(t)$ contain the message processes, $\phi_1(t)$ and $\phi_2(t)$ which are random phase fluctuation processes, we observe that the signals $\tilde{E}_{1c}(t) \exp(j\omega_1 t)$ and $\tilde{E}_{2c}(t) \exp(j\omega_2 t)$ are random narrow band optical processes centered around the two optical frequencies ω_1 and ω_2 respectively.

With $\tilde{E}_{1c}(t)$ and $\tilde{E}_{2c}(t)$ are given by

$$\tilde{E}_{1c}(t) = (p_1(t))^{1/2} \exp(jK_1 p_1(t) + j2K_1 p_2(t) + j\phi_1(t)) \tag{2.49}$$

$$\tilde{E}_{2c}(t) = (p_2(t))^{1/2} \exp(jK_2 p_2(t) + j2K_2 p_1(t) + j\phi_2(t)) \tag{2.50}$$

as shown earlier, the resulting photo current is given by

$$i(t) = \chi \, [\, E(t) \,]^2 \; (*) \; h_d(t) \,, \tag{2.51}$$

where χ is usually taken as 1 amp/watt. The resulting current could be written as

$$i(t) = p_1(t) + p_2(t) + 2\,\mathrm{Re}\,\{\, \tilde{E}_{1c}(t)\, \tilde{E}_{2c}(t)\, \{\exp(j2\pi\Delta\nu t)\,\} \qquad (2.52)$$

The total power spectrum of the photo current would be

$$S(f) = \mathcal{F}_T \left[\mathcal{E} \left(\overline{i(t+\tau)\, i^{'}(t)} \right) \right] . \qquad (2.53)$$

As seen from earlier discussions, the significant contribution to the cross power spectral density $S_x(f)$ is the power spectrum of the cross term.

$$S_x(f) = \mathcal{F}_T \left[\mathcal{E} \left\{ \overline{\mathrm{Re}\left(E_1(t+\tau)\, E_2^{*}(t+\tau) \right) \mathrm{Re}\left(E_1^{*}(t)\, E_2(t) \right)} \right\} \right] \qquad (2.54)$$

The main thrust of this thesis is to derive and evaluate eq. (2.53), the cross power spectrum under some specific cases.

After evaluting eq. (2.53), we can find the signal to interference ratio, SIR, as

$$\mathrm{SIR} = \frac{\text{(Total message power)}}{\text{(Interference power within in message bandwidth)}}$$

$$\mathrm{SIR} = \frac{\int S_1(f)\, df}{\int S_x(f)\, dt} \qquad (2.55)$$

where the integration of $S_x(f)$ is carried out over message bandwidth and, $S_1(f)$ is the received power spectrum of the message $p_1(t)$.

CHAPTER - 3

ANALYSIS AND DISCUSSION

3.1 THE BASIC STRUCTURE

The block diagram in Fig.3.1 is a simple two optical carrier system, with direct detection scheme, and the envelopes being sub-carrier multiplexed. Such a scheme is analysed, and the signal to interference ratio (SIR) parameter which is a measure of the effect of SPM and XPM is evaluated for 2 optical carrier system; the resultis generalized to systems with more than 2 optical carriers.

3.2 ASSUMPTIONS

In subsequent analysis the following assumptions were made

a) The only type of non-linear effects considered are the SPM and XPM.

b) The type of fiber is single mode.

c) The photo diode acts as an ideal low pass filter with a cut off frequency in the microwave region.

d) The effect of noise introduced at any stage of the system (Fig.3.1) is considered negligible.

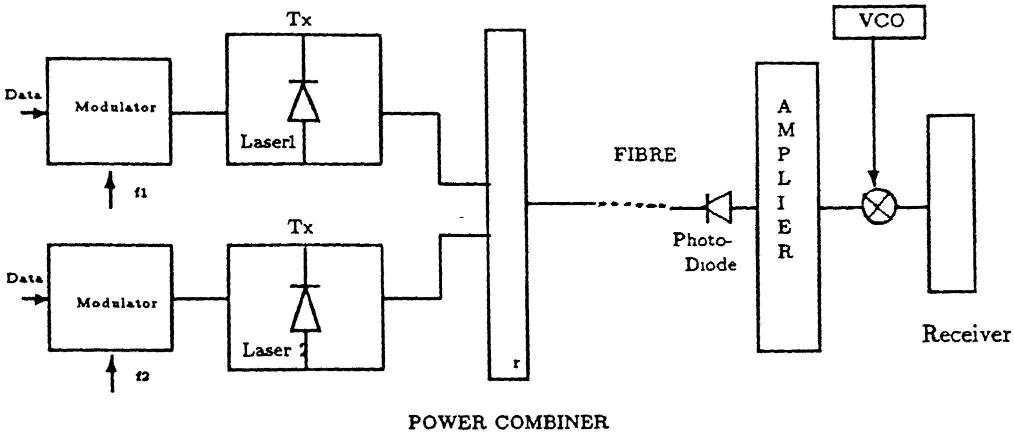


Figure 3.1: A Simple Two Carrier Light Wave System

3.3 CASE ONE

At first, we will consider the case of two sine waves at microwave frequencies f_1 and f_2 intensity modulating two optical carriers ν_1 and ν_2. The optical carrier being combined in a power combiner, launched in the optical fiber. At the receiver the combined signal is passed through a photo diode and a low noise amplifier, the two microwave frequencies f_1 and f_2 are recovered using microwave receivers whose bandwidths are centered around f_1 and f_2. Theoretical evaluation of SIR of such a case is presented below, wherein the effect of laser phase noise is considered.

Let $E_{1c}(t)$ and $E_{2c}(t)$ be the two complex electric fields with ν_1 and ν_2 being the optical carrier frequencies.

The combined electric field E(t)

$$E(t) = \mathrm{Re}\{E_{1c}(t)\} + \mathrm{Re}\{E_{2c}(t)\} \tag{3.1}$$

where

$$E_{1c}(t) = (p_1(t))^{1/2} \exp(j\nu_1 t + jK_1p_1(t) + 2jK_1p_2(t) + j\phi_1(t)) \tag{3.2}$$

$$E_{2c}(t) = (p_2(t))^{1/2} \exp(j\nu_2 t + jK_2p_2(t) + 2jK_2p_1(t) + j\phi_2(t)) \tag{3.3}$$

and,

$$K_1 \cong \frac{n_2\ \nu_1}{C\ A_{eff}\ \alpha} \quad \text{SPM factor 1} \tag{3.4}$$

$$K_2 \cong \frac{n_2 \nu_2}{C A_{eff} \alpha} \quad \text{SPM factor 2} \tag{3.5}$$

Refer to Section 2.1 for the definitions of K_1 and K_2. Furthermore,

$$p_1(t) = p_1(1 + A_1 \sin(\omega_1 t) \quad \text{with} \quad |A_1 \sin(\omega_1(t))| < 1 \tag{3.6}$$

$$p_2(t) = p_2(1 + A_2 \sin(\omega_2 t) \quad \text{with} \quad |A_2 \sin(\omega_2(t))| < 1 \tag{3.7}$$

and, p_1 and p_2 are average laser source powers,
A_1 and A_2 are constants and control the depth of modulation.

For the sake of simplicity we assume

$$A_1 = A_2 \quad \text{and} \quad p_1 = p_2$$

The incident intensity on the photo detector would be

$$[\, E(t)]^2 = (\mathrm{Re}\,\{\, E_{1c}(t)\,\} + \mathrm{Re}\,\{\, E_{2c}(t)\,\})^2 \tag{3.8}$$

$$\begin{aligned} = \frac{|E_{1c}(t)|^2}{2} + \frac{|E_{2c}(t)|^2}{2} &+ \mathrm{Re}\,\{\, E_{1c}(t)\, E_{2c}^{*}(t)\,\} \\ &+ \mathrm{Re}\,\{\, E_{1c}(t)\, E_{2c}(t)\,\} \\ &+ \mathrm{Re}\,\{\, E_{2c}(t)\, E_{2c}(t)\,\} \\ &+ \mathrm{Re}\,\{\, E_{1c}(t)\, E_{2c}(t)\,\} \end{aligned} \tag{3.9}$$

It can be easily shown that terms like $\mathrm{Re}\,\{\, E_{1c}(t)\, E_{1c}(t)\,\}$, $\mathrm{Re}\,\{\, E_{2c}(t)\, E_{2c}(t)\,\}$, $\mathrm{Re}\,\{\, E_{1c}(t)\, E_{2c}(t)\,\}$ have negligible frequency contents in the region of operation of the photo diode.

The above mentioned terms are discussed and analysed in Sec 3.2 under a more general case.

The term Re { $E_{1c}(t)\, E_{2c}^{*}(t)$ } is centered around $|\nu_1 - \nu_2|$, this would lie within the cut off frequency of the photo diode.

On the assumption that if ν_1 and ν_2 are closely spaced optical carriers such that $|\nu_1 - \nu_2| \cong 0$ then $K_1 \cong K_2$, we can write the term Re $\{E_{1c}(t)\, E_{2c}^{*}(t)\}$ after low pass filtering as

$$\mathrm{Re}\ \{\ E_{1c}(t)\ E_{2c}^{*}(t)\ \} \cong \mathrm{Re}\left\{ \sum_{n}\sum_{l} C_{nl} \ \exp\left[jn\omega_1(t) + jl\omega_2(t) + j\phi_1(t) - j\phi_2(t)\right]\right\} \tag{3.10}$$

The above result is derived in Appendix B.1.

Here the coefficients C_{nl} are determined by Bessel coefficients, $J_n(\beta_1)$ and $J_l(\beta_2)$, where β_1 and β_2 depend on SPM and XPM factors, average power of the laser sources, and the factor controlling the depth of modulation. Further information is given in Appendix B.1.

We know that the Fourier transform of the autocorrelation of eq. (3.10) would yield the cross power spectrum.

$$\mathrm{Re}\ (A_c)\ \ \mathrm{Re}\ (B_c) = 1/2\ \mathrm{Re}\ (AB_c) + 1/2\ \mathrm{Re}\ (AB_c^{*}) \tag{3.11}$$

Taking the autocorrelation of eq. (3.10) we get

$$\mathrm{Re}\left[\mathfrak{E}\left\{E_{1c}(t,\ t+\tau)\ E_{2c}^{*}(t,t+\tau)\right\}\right]\ \ \mathrm{Re}\left[\ \mathfrak{E}\left\{E_{1c}^{*}(t)\ E_{2c}(t)\right\}\right]$$

$$= 1/2\ \mathrm{Re}\left[\mathfrak{E}\left\{\sum_{n}\sum_{l}\sum_{n'}\sum_{l'} C_{nl}\ C_{n'l'}^{*}\ \exp\ (jn\omega_1\tau + (n-n')j\omega_1 t + l j\omega_2\tau\right.\right.$$

$$\left.\left. + (l - l')j\omega_2 t + j\phi_1(t) - j\phi_1(t+\tau) + j\phi_2(t+\tau) - j\phi_2(t)\ )\right\}\ \right]$$

$$+\ \ 1/2\ \mathrm{Re}\left[\mathfrak{E}\left\{\sum_{n}\sum_{l}\sum_{n'}\sum_{l'} C_{nl}\ C_{n'l'}^{*}\ \exp\ (jn\omega_1\tau + (n+n')j\omega_1 t + l j\omega_2\tau\right.\right.$$

$$\left.\left. + (l + l')j\omega_2 t + j\phi_1(t) + j\phi_1(t+\tau) - j\phi_2(t+\tau) - j\phi_2(t)\ )\right\}\ \right] \quad (3.12)$$

Since we are considering a deterministic signal, the expectation operator is only for phase noise processes $\phi_1(t)$ and $\phi_2(t)$ with the characteristic that, (refer to Section 2.2)

$$\mathfrak{E}\left\{\ \exp\ (j|\phi(t) - \phi(t+\tau)|)\ \right\} = \exp\ (-|\tau|\ \pi\Delta\nu) \quad (3.13)$$

$$\mathfrak{E}\left\{\ \exp\ (j|\phi(t) + \phi(t+\tau)|)\ \right\} = 0 \quad (3.14)$$

where $1/\tau_c = \Delta\nu$; $\Delta\nu$ represents the half power laser spectral width.

Using the result of eq. (3.13) and (3.14) in eq. (3.12) and using the notion of independence between the phase processes $\phi_1(t)$ and $\phi_2(t)$ eq. (3.12) can be reduced to

$$\mathrm{Re}\left[\mathfrak{F}\left\{E_{1c}(t,\ t+\tau)\ E^*_{2c}(t,t+\tau)\right\}\right] \quad \mathrm{Re}\left[\ \mathfrak{F}\left\{E^*_{1c}(t)\ E_{2c}(t)\right\}\right]$$

$$= \ 1/2\ \mathrm{Re}\left[\mathfrak{F}\ \left\{\sum_n\sum_l\sum_{n'}\sum_{l'}\ C_{nl}\ C^*_{n'l'}\ \exp\ (jn\omega_1\tau\ +\ (n-n')j\omega_1 t\ +\ l j\omega_2\tau\right.\right.$$

$$\left.\left. +\ (l\ -\ l')j\omega_2 t\ +\ j\phi_1(t)\ -\ j\phi_1(t+\tau)\ +\ j\phi_2(t+\tau)\ -\ j\phi_2(t)\ )\right\}\ \right] \qquad (3.15)$$

Since we are interested in the average power spectrum we perform the time average of eq. (3.15) before computing the Fourier transform. It is shown in Sec. 3.2 for a more general case that, the time averaging converts it into a double summation. Now eq. (3.15) becomes

$$\mathrm{Re}\left[\mathfrak{F}\left\{E_{1c}(t,\ t+\tau)\ E^*_{2c}(t,t+\tau)\right\}\right] \quad \mathrm{Re}\left[\ \mathfrak{F}\left\{E^*_{1c}(t)\ E_{2c}(t)\right\}\right]$$

$$= \ 1/2\ \mathrm{Re}\left\{\sum_n\sum_l\ C_{nl}\qquad \exp\ (jn\omega_1\tau\ +\ j\omega_2\tau)\ \exp\ (-|\tau|\ \ 2\pi(\Delta\nu_1+\Delta\nu_2)\ )\right\} \qquad (3.16)$$

We can easily compute the Fourier transform of eq. (3.16). This derivation is given in Appendix B.2.

$$S_X(f) \ = \ 1/4\sum_n\sum_l\ C_{nl}\ C^*_{nl}\ \left\{\frac{2}{(\pi\Delta\nu)}\left[\frac{1}{1+\left[\dfrac{2(f\ -\ f_{nl})}{\Delta\nu}\right]^2} + \frac{1}{1+\left[\dfrac{2(f\ -\ f_{nl})}{\Delta\nu}\right]^2}\right]\right\} \qquad (3.17)$$

where $f_{nl} = nf_1 + lf_2$

and, $f_1 = \omega_1/2\pi$ and $f_2 = \omega_2/2\pi$ are the two sub-carrier frequencies.

3.3.1 Expression for evaluation of SIR

The average power of the received single tone signal is $1/4(A_1)^2$. We assume that the microwave receiver bandwidth is bHz. Then the expression for SIR for the microwave receiver receiving the single tone signal at frequency f_1 is given by

$$\underset{(\text{at } f_1)}{\text{SIR}} = \frac{1/4\,(A_1)^2}{\int_{f_1-b/2}^{f_1+b/2} S_x(f)\,dt} \qquad (3.18)$$

Since the Lorentzian shaped spectrum has a smooth variation, we can approximate it by a constant over a small region of interest. Eq. (3.18) can now be written as

$$\underset{(\text{at } f_1)}{\text{SIR}} = \frac{1/4\,(A_1)^2}{S_x(f_1) \text{ x b Hz (B.W. of } R_x)} \qquad (3.19)$$

Example :

Let $f_1 = 2$ GHz and $f_2 = 2.01$ GHz

$p_1 = p_2 = 1$ mW, the average laser power

$\lambda_1 \cong \lambda_2 = 1.55\ \mu m$

Then

$K_1 = K_2 = 56.398\ W^{-1}$

where

$A_{eff} = 50\ \mu m^2$, $\alpha = 0.046\ km^{-1}$ (0.2 dB/km)

$A_1 = A_2 = 0.75$ (75 % modulation)

Bandwidth of receiver $\cong$ 5 KHz (centered at f_1)

$\Delta\nu_1 + \Delta\nu_2 \cong 50$ MHz , The total half power bandwidth of laser source I and laser source II added.

The coefficients, C_{nl}, were calculated and a computer program was written to evaluate the following equation,

$$\underset{(\text{at } f_1)}{\text{SIR(dB)}} = 10 \log \left[\frac{1/4\,(A_1^2)}{S_x(f_1) \times b \text{ Hz (B.W. of } R_x)} \right] \qquad (3.20)$$

It was found that

$\underset{\text{at } f_1}{\text{SIR (dB)}} = 44.18$

and

$\underset{\text{at } f_2}{\text{SIR (dB)}} = 44.18$

3.4 CASE TWO

In this section we consider the case of two data streams $m_1(t)$ and $m_2(t)$ both being BPSK modulated with sub-carrier frequencies of f_1 and f_2

$$m_1(t) = A_1 \sin(2\pi f_1 t + \alpha_1(t)) \qquad (3.21)$$

$$m_2(t) = A_2 \sin(2\pi f_2 t + \alpha_2(t)) \qquad (3.22)$$

where,

$$\alpha_1(t) = 2\pi f_d \sum_j a_j\, g(t - nT)$$

$$\alpha_2(t) = 2\pi f_d \cdot \sum_k a_k\, g(t - nT)$$

where,

f_d = phase modulation index

a_k, a_j = ± 1, binary data streams

$g(t)$ is unit rectangular pulse

T is the bit period

Now the modulated envelopes of the optical carrier frequencies ν_1 and ν_2 are

$$(p_1(t))^{1/2} = (p_1(1 + m_1(t))^{1/2} \tag{3.23}$$

$$(p_2(t))^{1/2} = (p_2(1 + m_2(t))^{1/2} \tag{3.24}$$

p_1 and p_2 are average laser powers.

The two electric fields $E_{1c}(t)$ and $E_{2c}(t)$ are combined in a power combiner and sent in the fiber. The incident intensity on the photo detector would be

$$[E(t)]^2 = [\mathrm{Re}\,\{ E_{1c}(t) \} + \mathrm{Re}\,\{ E_{2c}(t) \}]^2 \tag{3.25}$$

where

$$E_{1c}(t) = (p_1(t))^{1/2} \exp\,(j\nu_1 t + jK_1 p_1(t) + 2jK_1 p_2(t) + j\phi_1(t)) \tag{3.26}$$

$$E_{2c}(t) = (p_2(t))^{1/2} \exp\,(j\nu_2 t + jK_2 p_2(t) + 2jK_2 p_1(t) + j\phi_2(t)) \tag{3.27}$$

In the above equations, the symbols have their usual meanings as explained earlier.

After squaring operation we get

$$\begin{aligned}[E(t)]^2 = (p_1(t))/2 + (p_2(t))/2 &+ \mathrm{Re}\,\{\, E_{1c}(t)\, E_{2c}^{*}(t) \,\} \\ &+ \mathrm{Re}\,\{\, E_{1c}(t)\, E_{2c}(t) \,\} \\ &+ \mathrm{Re}\,\{\, E_{2c}(t)\, E_{2c}(t) \,\} \\ &+ \mathrm{Re}\,\{\, E_{1c}(t)\, E_{2c}(t) \,\}\end{aligned} \tag{3.28}$$

Consider the term Re { $E_{1c}(t)$ $E_{1c}(t)$ }. By substituting for $E_{1c}(t)$ we get

$$\mathrm{Re}\,\left\{E_{1c}(t)\, E_{1c}(t)\right\}$$

$$= \mathrm{Re}\,\{\, (p_1(t)) \exp\,(2j\nu_1 t + 2jK_1 p_1(t) + 4jK_1 p_2(t) + 2j\phi_1(t)) \,\} \tag{3.29}$$

$$\begin{aligned}= \mathrm{Re}\,\{\, p_o(1 + A_1 \sin(\omega_1 t + \alpha_1(t)) \exp\,(2j\nu_1 t + 2j\phi_1(t) + \phi_o \\ + \beta_1 \sin(\omega_1 t + \alpha_1(t)) + \beta_2 \sin(\omega_2 t + \alpha_2(t)) \,\}\end{aligned} \tag{3.30}$$

$$\begin{aligned}= \mathrm{Re}\Big\{p_o(1+A_1 \sin(\omega_1 t+\alpha_1(t))\Big[\sum_n \sum_l J_n(\beta_1) J_l(\beta_2) \exp\,(2j\nu_1 t+2j\phi_1(t)+j\phi_o \\ + n(\omega_1 t + \alpha_1(t)) + l(\omega_2 t + \alpha_2(t))\Big]\Big\}\end{aligned} \tag{3.31}$$

By substituting,

$$\sin(\omega_1 t + \alpha_1(t)) = \frac{-j}{2}\left[\exp[j\omega_1 t + \alpha_1(t)] - \exp[-j\omega_1 t - j\alpha_1(t)]\right]$$

equation (3.31) can be written as

$$= \mathrm{Re}\left\{\sum\sum C_{nl} \exp\left(2j\nu_1 t + 2j\phi_1(t) + j\phi_o + jn(\omega_1 t + n\alpha_1(t)) + j(l\omega_1 t + l\alpha_2(t))\right)\right\} \qquad (3.32)$$

where

$$C_{nl} = \left[J_n(\beta_1)\, J_n(\beta) - \frac{jp_o A_1}{4}\left[J_{n-1}(\beta_1)\, J_1(\beta_2) + J_{n+1}(\beta_1)\, J_1(\beta_2)\right]\right] \qquad (3.33)$$

(refer to Appendix B.1 for further details).

For $|n| > 2$ and $|l| > 2$ the coefficients C_{nl} go to zero.

Hence it is evident from eq. (3.32) that the contribution of frequency to the region of operation of the photo diode is negligible and therefore can be neglected. Similarly it can be shown that Re { $E_{1c}(t)\, E_{2c}(t)$ } and Re { $E_{1c}(t)\, E_{2c}(t)$ } have also negligible frequency content within the cut off frequency of photo diode.

Now consider the term

Re { $E_{1c}(t)\, E_{2c}^{*}(t)$ } upon substituting for $E_{1c}(t)$ and $E_{2c}(t)$ we get

$$\mathrm{Re}\ \left\{E_{1c}(t)\ E_{2c}^{*}(t)\right\}$$

$$= \mathrm{Re}\ \left\{(p_1(t)\ p_2(t))^{1/2}\exp\left[(j\nu_1 - j\nu_2)\ t + j\phi_1(t) - j\phi_2(t)\right.\right.$$

$$\left.\left. + jK_1p_1(t) + 2jK_1p_2(t) - jK_2p_2(t) - 2jK_2p_1(t)\right]\right\}$$

(3.34)

where

$p_1(t) = (p_o(1 + A_1\sin(\omega_1 t + \alpha_1(t))))$ and $|A_1\ \sin(\omega_1 t + \alpha_1(t))| < 1$

$p_2(t) = (p_o(1 + A_2\sin(\omega_2 t + \alpha_2(t))))$ and $|A_2\ \sin(\omega_2 t + \alpha_2(t))| < 1$

(3.35)

The assumption that ν_1 and ν_2 are closely spaced, and so $|\nu_1 - \nu_2| \cong 0$ implies $K_1 \cong K_2$ with this assumption. When eq. (3.35) is substituted in eq. (3.34) we get

$$\mathrm{Re}\ \left\{E_{1c}(t)\ E_{2c}^{*}(t)\right\}$$

$$= \mathrm{Re}\left\{\left[[p_o(1 + A_1\ \sin(\omega_1 t + \alpha_1(t)))][p_o(1 + A_2\ \sin(\omega_2 t + \alpha_2(t)))]\right]^{1/2}\right.$$

$$\exp(-jK_1p_oA_1\ \sin(\omega_1 t + \alpha_1(t)) + jK_1p_oA_2\ \sin(\omega_1 t + \alpha_2(t))$$

$$\left.\exp\ (j\phi_1(t) - j\phi_2(t))\right\} \qquad (3.36)$$

By taking the approximation that

$$(1 + x)^{1/2} = 1 + x/2 \quad ; \quad |x| < 1$$

The envelope of eq. (3.36) can be written as

$$(p_o(1 + A_1 \sin(\omega_1 t + \alpha_1(t)))) \; (p_o(1 + A_2 \sin(\omega_2 t + \alpha_2(t))))$$

$$+ A_1 A_2 \sin(\omega_1 t + \alpha_1(t)) \sin(\omega_2 t + \alpha_2(t)) \quad (3.37)$$

The complex phase part of eq. (3.36) can be expressed in terms of Bessel's function [6] that is

$$\exp\left\{-jK_1 p_o A_1 \sin(\omega_1 t + \alpha_1(t)) + jK_1 p_o A_2 \sin(\omega_2 t + \alpha_2(t))\right.$$

$$\left.(j\phi_1(t) - j\phi_2(t)\right\} \quad (3.38)$$

$$= \sum \sum J_n(\beta_1) \, J_l(\beta_2)(-1)^n \exp\,(jn\,[\,\omega_1 t + \alpha_1(t)\,] + jl\,[\omega_2 t + \alpha_2(t)]$$

$$+ j\phi_1(t) - j\phi_2(t)) \quad (3.39)$$

where

$$\beta_1 = K_1 p_o A_1 \text{ and } \beta_2 = K_1 p_o A_2$$

On substituting the envelope part approximation, that is, eq. (3.37) and the phase part expression in eq. (3.39) and carrying out the analysis similar to that shown in Appendix B.1, we can write eq. (3.36) as

$$\mathrm{Re}\left\{E_{1c}(t)\, E^*_{2c}(t)\right\}$$

$$= \mathrm{Re}\left\{\sum \sum C_{nl} \exp\,(jn\,[\omega_1 t + \alpha_1(t)] + jl\,[\omega_2 t + \alpha_2(t)]\right.$$

$$\left.+ j\phi_1(t) - j\phi_2(t))\right\} \quad (3.40)$$

The resulting photo current can now be expressed as

$$i(t) = \frac{p_1(t)}{2} + \frac{p_2(t)}{2} + \mathrm{Re}\left\{ \sum\sum C_{nl} \exp\,(jn\,[\omega_1 t + \alpha_1(t)]\right.$$

$$\left. + jl\,[\omega_2 t + \alpha_2(t)] + j\phi_1(t) - j\phi_2(t))\right\} \text{ (cross term)} \qquad (3.41)$$

To calculate the cross power spectral density we have to calculate the cross power spectrum of eq. (3.12). The autocorrelation of the cross term is

$$1/2\ \mathrm{Re}\left[\mathcal{E} \left\{ \sum_n \sum_l \sum_{n'} \sum_{l'} C_{nl}\, C^*_{n'l'} \exp\,(jn\omega_1\tau + (n-n')j\omega_1 t + l j\omega_2\tau \right.\right.$$

$$+ (l - l')j\omega_2 t + jn\alpha_1(t+\tau) - jn'\alpha_1(t) + jl\alpha_2(t+\tau) - jl'\alpha_2(t) +$$

$$\left.\left. j\phi_1(t) - j\phi_1(t+\tau) + j\phi_2(t+\tau) - j\phi_2(t)\ )\right\}\right]$$

$$+ 1/2\ \mathrm{Re}\left[\mathcal{E} \left\{ \sum_n \sum_l \sum_{n'} \sum_{l'} C_{nl}\, C^*_{n'l'} \exp\,(jn\omega_1\tau + (n+n')j\omega_1 t + l j\omega_2\tau \right.\right.$$

$$+ (l + l')j\omega_2 t + jn\alpha_1(t+\tau) + jn'\alpha_1(t) + jl\alpha_2(t+\tau) + jl'\alpha_2(t) +$$

$$\left.\left. j\phi_1(t) + j\phi_1(t+\tau) - j\phi_2(t+\tau) - j\phi_2(t)\ )\right\}\right] \qquad (3.42)$$

We define

$$\tilde{y}_{nn}(t,t+\tau) = \exp\,(jn\alpha_1(t+\tau) - jn'\alpha_1(t)) \qquad (3.43)$$

and

$$\tilde{y}_{ll}(t,t+\tau) = \exp\,(jl\alpha_2(t+\tau) - jl'\alpha_2(t)) \qquad (3.44)$$

Since the two message sources $\alpha_1(t)$ and $\alpha_2(t)$ are independent, $\tilde{y}_{nn}(t,t+\tau)$, $\tilde{y}_{ll}(t,t+\tau)$ are also independent stochastic processes. $\phi_1(t)$ and $\phi_2(t)$ are also two independent non-stationary zero mean stochastic processes. It is to be noted that $\tilde{y}_{nn}(t,t+\tau)$,

$\tilde{y}_{11}(t,t+\tau)$, $\phi_1(t)$, $\phi_2(t)$ are all independent stochastic processes. By invoking the property of independence we can write eq. (3.42) as

$$1/2\ \mathrm{Re}\Big[\sum_n\sum_l\sum_{n'}\sum_{l'} C_{nl}\, C^*_{n'l}\ \exp\,(jn\omega_1\tau + (n-n')j\omega_1 t + l j\omega_2\tau$$

$$+ (l - l')j\omega_2 t) * \left\{\tilde{y}_{nn}(t,t+\tau)\right\} * \left\{\tilde{y}_{ll}(t,t+\tau)\right\}$$

$$* \left\{\exp(j\phi_1(t) - j\phi_1(t+\tau))\right\} * \left\{\exp(j\phi_2(t+\tau)-j\phi_2(t))\right\}\Big]\ \text{(first term)}$$

$$+\ 1/2\ \mathrm{Re}\Big[\sum_n\sum_l\sum_{n'}\sum_{l'} C_{nl}\, C^*_{n'l}\ \exp\,(jn\omega_1\tau + (n-n')j\omega_1 t + l j\omega_2\tau$$

$$+ (l - l')j\omega_2 t) * \left\{\exp\,(jn\alpha_1(t+\tau)+jn'\alpha_1(t))\right\}$$

$$*\left\{\exp\,(jl\alpha_2(t+\tau) + jl'\alpha_2(t))\right\}$$

$$*\left\{\exp\,(j\phi_1(t)+j\phi_1(t+\tau))\right\} * \left\{\exp(-j\phi_2(t+\tau) - j\phi_2(t))\right\}\Big]\ \text{(second term)} \tag{3.45}$$

By the aplication of the result of eq. (2.14) the second term of eq. (3.45) goes to zero and only the first term remains.

Since we are interested in the average power spectral density we have to compute the time average autocorrelation function, time average is over variable t.

Since $\tilde{y}_{nn}(t,t+\tau)$, $\tilde{y}_{ll}(t,t+\tau)$, $\phi_1(t)$, $\phi_2(t)$ are all independent stochastic processes, the time average of the

first term of eq. (3.45) is the product of the time averages of each of them, and they are time separable [5]. That is,

$$\langle \Gamma_{E1} \Gamma_{E2} \rangle = \langle \Gamma_{E1} \rangle \langle \Gamma_{E2} \rangle \tag{3.46}$$

where $\langle\ \rangle$ denotes the time averaging operation.

After doing the time average of the first term of eq. (3.45), we get

$$\begin{aligned} 1/2 \ \mathrm{Re} \Bigg[\sum_{n} \sum_{l} \sum_{n'} \sum_{l'} C_{nl} C^{*}_{n'l'} \ \langle \mathcal{E} \left\{ \tilde{y}_{nn}(t,t+\tau) \right\} \rangle \langle \mathcal{E} \left\{ \tilde{y}_{ll}(t,t+\tau) \right\} \rangle \\ \langle \mathcal{E} \left\{ \exp\left(j\phi_1(t) - j\phi_1(t+\tau) + jn\omega_1\tau + (n-n')j\omega_1 t \right) \right\} \rangle \\ \langle \mathcal{E} \left\{ \exp\left(j\phi_2(t+\tau) - j\phi_2(t) + jl\omega_2\tau + (l-l')j\omega_2 t \right) \right\} \rangle \Bigg] \end{aligned} \tag{3.47}$$

We know that

$$\mathcal{E} \left\{ \exp\left(j\phi_1(t+\tau) - j\phi_1(t) \right) \right\} = \exp\left(-\pi|\tau|/\tau_{c1}\right) \tag{3.48}$$

and $$\mathcal{E} \left\{ \exp\left(j|\phi_2(t) - j\phi_2(t+\tau))| \right) \right\} = \exp\left(-\pi|\tau|/\tau_{c2}\right) \tag{3.49}$$

These results are derived in Appendix A.1.

By applying the results of eq. (3.48) and eq. (3.46) it follows that

$$\begin{aligned} \langle \mathcal{E} \int \exp\left(j(\phi_1(t+\tau) - \phi_1(t)) - j(n-n')\omega_1 t \right) \rangle = \\ \langle \exp(-\pi|\tau|/\tau_{c1}) \int \exp(j(n-n')\,\omega_1 t)\, dt \end{aligned} \tag{3.50}$$

Since we know that

$$\frac{1}{T_1}\int_{-T_1/2}^{+T_1/2} \exp(j(n-n')\,\omega_1 t)\,dt = 0 \;;\; \text{for } n \neq n'$$
$$= 1 \;;\; \text{for } n = n'$$

$$\frac{1}{T_2}\int_{-T_2/2}^{+T_2/2} \exp(j(l-l')\,\omega_2 t)\,dt = 0 \;;\; \text{for } l \neq l'$$
$$= 1 \;;\; \text{for } l = l'$$

where $T_1 = \frac{2\pi}{\omega_1}$ and $T_2 = \frac{2\pi}{\omega_2}$.

The eq. (3.47) exists only for n= n', and l = l' and by substituting the same eq. (3.47) reduces to double summation

$$= 1/2\ \text{Re}\left\{ \sum_n \sum_l C_{nl}\, C^*_{nl}\, \psi_1(\tau)\, \psi_2(\tau)\, \exp(-|\tau|/\tau_{c1})\, \exp(-|\tau|\tau_{c2}) \right.$$

$$\left. \exp\,(jn\omega_1\tau + jl\omega_2\tau) \right\} \tag{3.51}$$

It is shown in [7] that the time average autocorrelation of the following are functions of τ only.

$$< \mathcal{E} \left\{\exp\,(jn(\alpha_1(t+\tau) - \alpha_1(t))\right\} > \;=\; \psi_1(\tau) \tag{3.52}$$

$$< \mathcal{E} \left\{\exp\,(jl(\alpha_2(t+\tau) - \alpha_2(t))\right\} > \;=\; \psi_2(\tau) \tag{3.53}$$

The fourier transform of the eq. (3.51) is

$$S_x(f) = 1/2\ \text{Re}\left\{ \sum_n \sum_l |C_{nl}|^2\, S_1(f)\ (*)\ S_2(f)\ (*) \right.$$

$$S_{1or1}(f)\ (*)\ S_{1or}(f)\ (*)\ \delta\,(f - (nf_1 + lf_2)) \tag{3.54}$$

$$= 1/4 \sum_n \sum_l |C_{nl}|^2 \left\{S_1(.) * S_2(.) * S_{1or1}(.) * S_{1or2}(.)\right\}(f-(n\omega_1+l\omega_2))$$

$$+\sum \sum |C_{nl}|^2 \left\{S_1(.) (*) S_2(.) (*) S_{1or1}(.) (*) S_{1or2}(.)\right\}(f+(n\omega_1+l\omega_2))$$

$$(3.55)$$

Under the assumption that the half power bandwidths of the laser source 1 and laser source 2 are much greater than the message bandwidth, the equation of $S_x(f)$ reduces to

$$S_X(f) = 1/2 \sum_n \sum_l |C_{nl}|^2 \left[\frac{2}{\pi\Delta\nu} \left[\frac{1}{1 + \left[\frac{2(f - f_{nl})}{\Delta\nu}\right]^2} + \frac{1}{1 + \left[\frac{2(f - f_{nl})}{\Delta\nu}\right]^2} \right] \right] \qquad (3.56)$$

Since a microwave receiver is used whose bandwidth is equal to message bandwidth at the O/P of microwave receiver, the SIR, say for message one would be

$$\underset{\text{one}}{\overset{\text{SIR}}{\text{message}}} = \frac{1/4\ (m_1(t))^2}{S_x(f_1) \text{ x bandwidth of message one}}$$

and for message two would be

$$\underset{\text{two}}{\overset{\text{SIR}}{\text{message}}} = \frac{1/4\ (m_2(t))^2}{S_x(f_2) \text{ x bandwidth of message two}}$$

This result can easily be genralized to more than two carriers, consider 3 optical carriers, each of them have messages

$p_1(t)$, $p_2(t)$ and $p_3(t)$ in sub-carrier centered at f_1, f_2 and f_3, then the resulting field incident on photo detector would be

$$[E(t)]^2 = \frac{p_1(t)}{2} + \frac{p_2(t)}{2} + \frac{p_3(t)}{2} + \text{cross term} \qquad (3.57)$$

Assuming that the optical frequencies are closely spaced, i.e., $\nu_1 \cong \nu_2 \cong \nu_3$. The cross terms that would contribute to the cross power spectrum will be

$$\mathrm{Re}\left\{E_{1c}(t)\ E^*_{2c}(t)\right\}, \quad \mathrm{Re}\left\{E_{2c}(t)\ E^*_{3c}(t)\right\}, \quad \mathrm{Re}\left\{E_{1c}(t)\ E^*_{3c}(t)\right\}$$

each of them can be approximated by a bessel function, as shown in Appendix B.1. Theresulting cross power spectrum can be written as

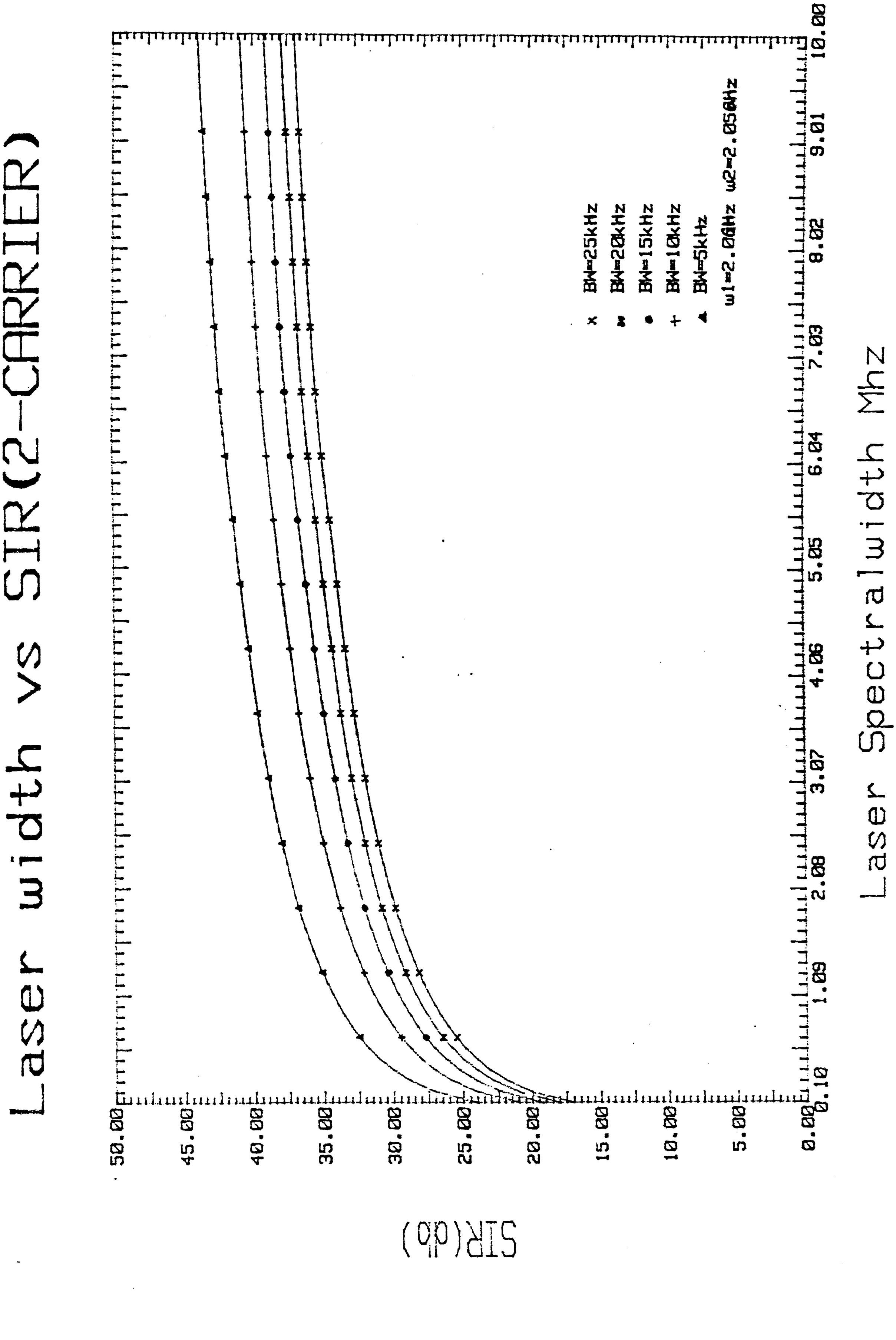
$$S_x(f) = S_{12x}(f) + S_{23x}(f) + S_{31x}(f)$$

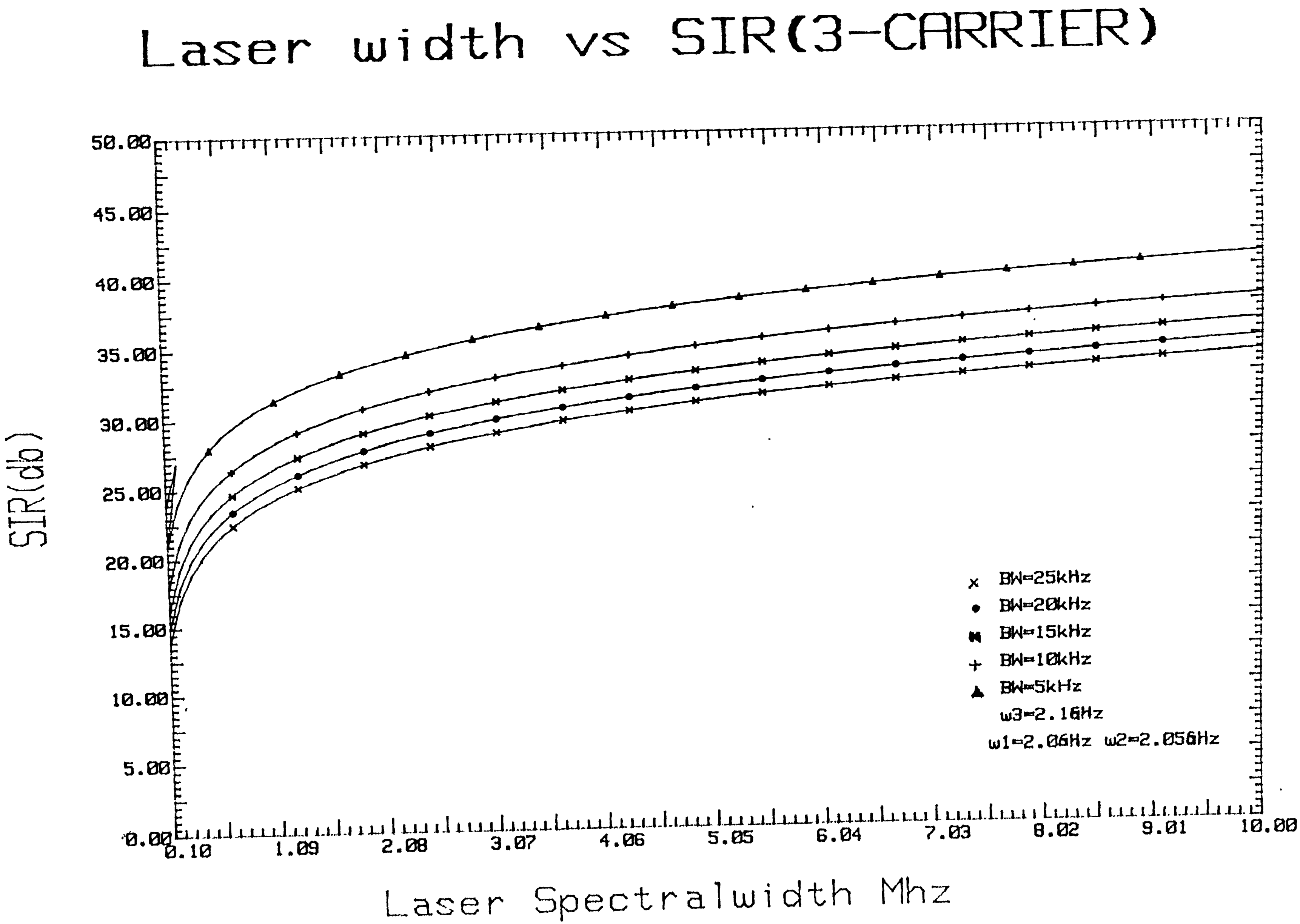
where

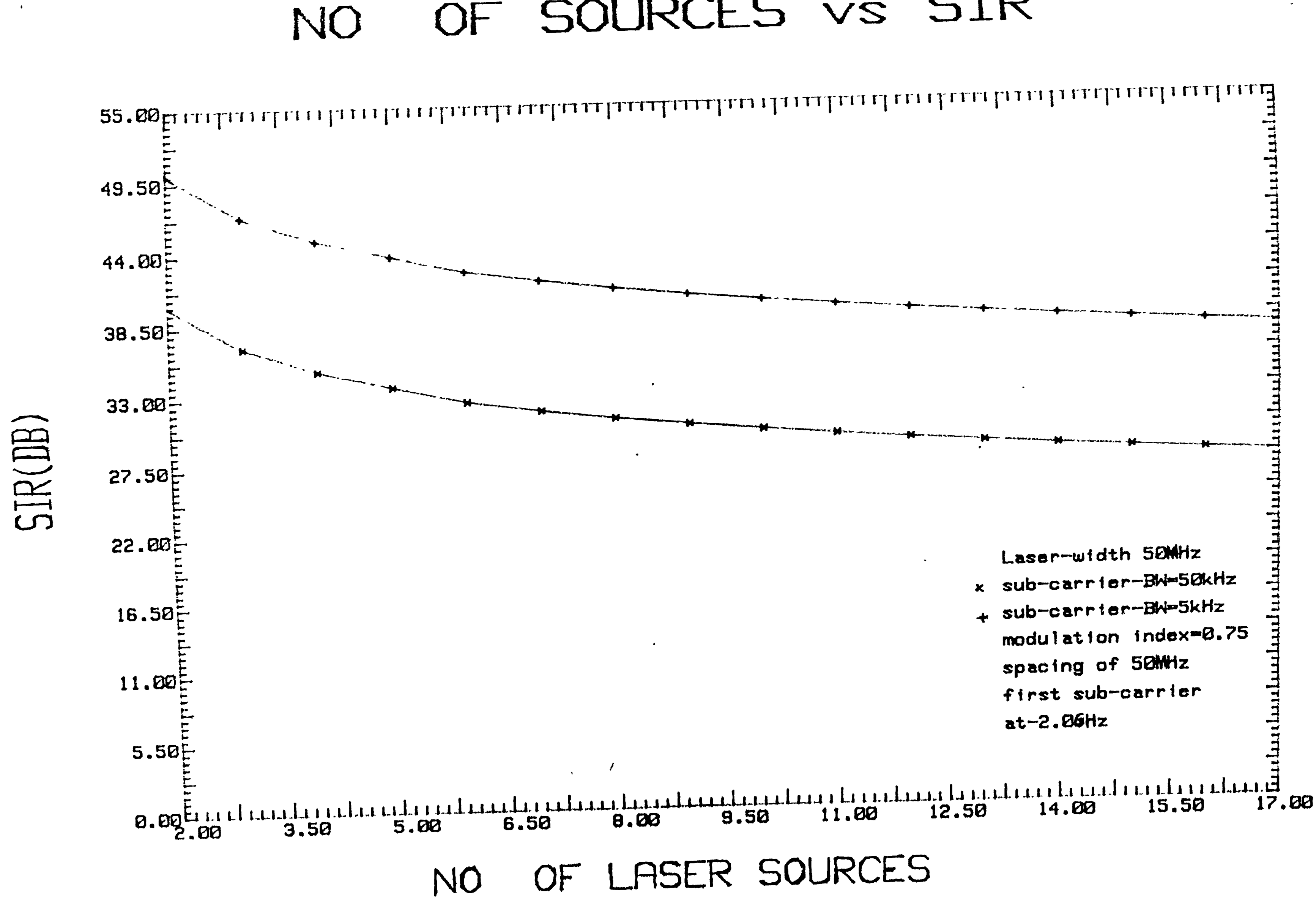
$S_{12x}(f) + S_{23x}(f) + S_{31x}(f)$ are the cross power spectra due to the factors Re ($E_{1c}(t)\ E^*_{2c}(t)$), Re ($E_{2c}(t)\ E^*_{3c}(t)$), and Re ($E_{1c}(t)\ E^*_{3c}(t)$)respectively, where $S_{12x}(f)$, $S_{23x}(f)$ and $S_{31x}(f)$ are similar to the power spectrum expressed in eq. (3.56)

A graph is drawn with the laser spectral width variation as the abscissa and the SIR variation as the ordinate. For a 2 carrier (Fig. 3.2) and 3 carrier (Fig. 3.3) case the following were the values of different parameters

$$\lambda_3 \cong \lambda_1 \cong \lambda_1 = 1.55\ \mu m$$
$$K_3 = K_1 = K_2 = 53.45\ W^{-1}$$
$$A_1 = A_2 = 0.75 \quad (75\%\ \text{modulation})$$


Laser width vs SIR(2-CARRIER)
SIR(db)
Laser Spectralwidth Mhz
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00
40.00
45.00
50.00
0.10
1.09
2.08
3.07
4.06
5.05
6.04
7.03
8.02
9.01
10.00
× BW=25kHz
BW=20kHz
• BW=15kHz
+ BW=10kHz
▲ BW=5kHz
w1=2.00Hz w2=2.050Hz


Laser width vs SIR(3-CARRIER)
SIR(db)
50.00
45.00
40.00
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
0.10
1.09
2.08
3.07
4.06
5.05
6.04
7.03
8.02
9.01
10.00
Laser Spectralwidth Mhz
BW=25kHz
BW=20kHz
BW=15kHz
BW=10kHz
BW=5kHz
w3=2.1GHz
w1=2.06Hz w2=2.056Hz


NO OF SOURCES VS SIR
SIR(DB)
55.00
49.50
44.00
38.50
33.00
27.50
22.00
16.50
11.00
5.50
0.00
2.00
3.50
5.00
6.50
8.00
9.50
11.00
12.50
14.00
15.50
17.00
Laser-width 50MHz
x sub-carrier-BW=50kHz
+ sub-carrier-BW=5kHz
modulation index=0.75
spacing of 50MHz
first sub-carrier
at-2.0GHz
NO OF LASER SOURCES

Average laser power of all lasers is taken to be 1 mW. The half power laser width $\Delta\nu$ is varied between 100 KHz - 10 MHz, the sub-carier frequencies were centered at $f_1 = 2.0$ GHz, $f_2 = 2.01$ GHz and $f_3 = 2.1$ GHz with a bandwidth increasing from 5 KHz to 25 KHz in steps of 5 KHz.

As seen from the above graph, as $\Delta\nu$ increases SIR increases. This can be explained by the fact that the average laser power being constant, increasing $\Delta\nu$ decreases the integral,

$$\int_{f-B/2}^{f+B/2} S_x(f)\, df$$

But as $\Delta\nu$ is increased more, the strength of interference power contributed by other terms increases and, therefore SIR variation is not significant.

Figure 3.4 shows the variation of the number of laser sources used in the system with SIR As is evident from the figure the SIR decreases with the number of laser sources increases. The spacing between sub-carriers was 50 MHz with the first sub-carrier at 2 GHz. The modulation index was 0.75 (75% modualtion). The SPM factor was equal to 53.95 W^{-1} and laser width was kept at 50 MHz.

CHAPTER - 4

RESULTS AND CONCLUSIONS

It was noted that the interference noise limits the maximum number of optical carriers in an intensity modulated sub-carrier multiplexed system. The amount of interference power increases when the bandwidth of the message modulating the sub-carriers were increased. Hence the number of optical carrier used reduces considerably.

The variations of the SPM and XPM factors for a fixed laser power did not considerably affect the SIR. There was a variation of about + 0.5 dB as the SPM and XPM factors were increased. It can also be seen that the increase in the laser power, would increase the coefficients, C_{nl}, having significant contribution to the cross power spectral density, which would be detrimental to SIR. It was also noted that if the spacing between the carriers were reduced, the SIR did not deviate from ± 1.0 dB limits. The present thesis has considered only the cases when the message bandwidth is much smaller than the half power laser spectral width.

The derivations and the consequent results were obtained under the assumptions that the optical carriers were closely spaced. This led to certain simplifications in derivations. An interesting problem would be to consider optical carriers sparsely placed. It would also be significant to consider the effect of the message bandwidth in the derivation of the PSD.

APPENDIX - A.1

To prove

$$\mathcal{E}\left\{ \exp(j(\phi(t+\tau) - \phi(t))\right\} = \exp\left(-|\tau|/\tau c\right) \tag{A.1.1}$$

We state a standard result [10]

$$\int_{-\infty}^{+\infty} \exp(-a^2b^2) \cos (bx)\, dx = \frac{\sqrt{\pi}}{a} \exp\left[\frac{-b^2}{2a^2}\right] \tag{A.1.2}$$

Since $\phi(t+\tau) - \phi(t)$ has a gaussian distributed random variable with zero mean and whose variance is $2\pi|\tau|/\tau c$.

We know that if x is a random variable with a density function p(x) then the expectation of g(x), which is a function of x, is given by [8]

$$\mathcal{E}\left\{ g(x) \right\} = \int_{-\infty}^{+\infty} g(x)\, p(x)\, dx$$

Using above equation we get

$$\mathcal{E}\left\{ \exp(j(\phi(t+\tau) - \phi(t))\right\}$$

$$= \int_{-\infty}^{+\infty} \frac{\exp(j\phi)\exp(\phi^2/2\sigma^2)}{\sqrt{2\pi}\sigma}\, d\phi = \int_{-\infty}^{+\infty} \frac{\cos\phi \exp(-\phi^2/2\sigma^2)}{\sqrt{2\pi}\sigma}\, d\phi$$

$$+ j\int_{-\infty}^{+\infty} \frac{\sin\phi \exp(-\phi^2/2\sigma^2)}{\sqrt{2\pi}\sigma}\, d\phi$$

Since sin ϕ is an odd function and $(\exp(-\phi^2/2\sigma^2)/\sqrt{2\pi}\sigma)$ is an even function the second integral becomes zero and we have to evaluate the first term only.

$$\therefore \int_{-\infty}^{+\infty} \frac{(\cos\phi)\exp(-\phi^2/2\sigma^2)}{\sqrt{2\pi}\sigma}\, d\phi = \frac{(2a^2)^{1/2}}{\sqrt{2\pi}\sigma} \int_{-\infty}^{+\infty} \cos(\phi)\exp(-a^2x^2)\, d\phi$$

where $a^2 = \dfrac{1}{2\sigma^2}$, implies $\sigma = \dfrac{1}{(2a^2)^{1/2}}$

$$\therefore \text{ the integral } = \frac{\sqrt{2}\ a \times \sqrt{\pi}}{\sqrt{2\pi}\qquad a} \exp\left(\frac{-2\sigma^2}{4}\right)$$

$$= \exp(-\sigma^2/2)$$

$$= \exp\left(\frac{-\pi|\tau|}{\tau c}\right)$$

APPENDIX - A.2

To show that

$$\mathcal{F}_T \left\{ \exp\ (+j\omega_o\tau)\ \exp\ (-\pi|\tau|\ /\tau c) \right\}$$

$$= \frac{(2(\pi\Delta\nu))^{-1}}{\left[1 + \left[\frac{2\pi(f-t_o)}{\pi\Delta\nu} \right]^2 \right]}$$

We know that the following are transform pairs [9]

$$\exp\ (-|\tau|) \qquad \frac{2}{1+(2\pi f)^2}$$

and the following properties are also known [9].

if $\quad g(\tau) \rightleftharpoons G(f)$

then $\quad g(a\tau) \rightleftharpoons \frac{1}{|a|}\ G\ (f/a)$ - property 1

and also $\quad g(t)\ e^{j2\pi f_o t} \rightleftharpoons G(f - f_o)$ - property 2

where $\quad a = \pi/\tau c$

Applying the transform pair and using the above properties

$$\mathcal{F}_T \left[\exp\ (-\pi|\tau|\ /\tau c) \right]$$

$$= \frac{2(\pi\Delta\nu)^{-1}}{\left[1 + \left[\frac{2\pi f}{\pi\Delta\nu} \right]^2 \right.}$$

and from property 2 we have

$$\mathcal{F}_T \left[\exp\,(+j\omega_o\tau)\ \exp\,(-\pi|\tau|/\tau c) \right]$$

$$= \frac{A\ (2(\pi\Delta\nu)^{-1}}{\left[\ 1\ +\left[\ \frac{2\pi(f-f_o)}{\pi\Delta\nu}\right]^2\right.}$$

where $\omega_o\ =\ 2\pi f_o$

and $1/\tau_c\ =\ \Delta v$; half source laser spectral width.

APPENDIX - A.3

To prove that

$$\mathcal{E}\left\{\exp\left[j(2\phi(t+\tau))\right]\right\} = 0 \text{ , for sufficiently large t.}$$

$\phi(t)$ is usually modelled as a continuous path Brownian motion process with zero mean [11]. The random variable for the noise process $\phi(t)$ is assumed to have a gaussian distribution [7]. The variance of the gaussian random variables, under such condition, increases with time.

Let $\mu(t_1)$ denote the instantaneous variance at time t_1.

By definition $\mathcal{E}\left\{\exp\left[(j(2(\phi(1+\tau))\right]\right\} =$ (A.3.1)

$$= \int_{-\infty}^{+\infty} \frac{\exp(2j\phi)\exp(-\phi^2/2\sigma_1^2)}{\sqrt{2\pi}\,\sigma_1} d\phi$$

where $\sigma_1^2 = \mu(t_1)$

The value of above integral reduces to

$$\int_{-\infty}^{+\infty} \frac{\cos(2\phi)\exp(-\phi^2/2\sigma_1^2)}{\sqrt{2\pi}\,\sigma_1} = \exp(-2\sigma_1^2)$$

For sufficiently large t the value of $\exp(-2\sigma_1^2)$ goes to zero, and hence the value of the eq. (A.3.1) goes to zero.

APPENDIX - B.1

Consider the term Re ($E_{1c}(t)\, E^{*}_{2c}(t)$)

here

$$E_{1c}(t) = (p_1(t))^{1/2} \exp\,(j\nu_1 t + jK_1 p_1(t) + 2jK_1 p_2(t) + j\phi_1(t))$$

and

$$E_{2c}(t) = (p_2(t))^{1/2} \exp\,(j\nu_2 t + jK_2 p_2(t) + 2jK_2 p_1(t) + j\phi_2(t))$$

where

$$p_1(t) \;=\; p_1(1 + A_1 \sin(\omega_1 t)) \quad \text{with} \quad |A_1 \sin(\omega_1(t))| < 1$$

and

$$p_2(t) \;=\; p_2(1 + A_2 \sin(\omega_2 t)) \quad \text{with} \quad |A_2 \sin(\omega_2(t))| < 1$$

p_1 and p_2 are average laser source powers,
A_1 and A_2 are constants and control the depth of modulation.

On the assumption that ν_1 and ν_2 are closely spaced optical carriers such that $\nu_1 - \nu_2 \cong 0$, then $K_1 = K_2$, we can write the term

$\mathrm{Re}\ \left\{E_{1c}(t)\ E^{*}_{2c}(t)\right\}$ as

$$\mathrm{Re}\ \left\{E_{1c}(t)\ E^{*}_{2c}(t)\right\}$$

$$= \mathrm{Re}\Big\{\Big[[p_o(1 + A_1 \sin(\omega_1 t))][p_o(1 + A_2 \sin(\omega_2 t))]\Big]^{1/2}$$

$$\exp(-jK_1 p_o A_1 \sin(\omega_1 t) + jK_1 p_o A_2 \sin(\omega_2 t))$$

$$\exp\,(j\phi_1(t) - j\phi_2(t))\Big\} \qquad \text{(B.1.1)}$$

where p_o is the average laser source power of both lasers.

By taking approximation that

$$(1+x)^{1/2} = (1+x/2) \quad \text{for} \quad |x| < 1$$

The envelope of the eq. (B.1.1) can be written as

$$= p_o^2 \ (1+(A_1/2)\sin(\omega_1 t))\ (1+(A_2/2)\sin(\omega_2 t)) \tag{B.1.2}$$

$$= p_o^2 \ (1+(A_1/2)\sin(\omega_1 t)+(A_2/2)\sin(\omega_2 t) + A_1A_2/8\Big[\cos((\omega_1-\omega_2)t) - \cos((\omega_1+\omega_2)t)\Big] \tag{B.1.3}$$

and the phase part of eq. (B.1.1) is given by [6].

$$\exp\left\{-j\beta_1\sin(\omega_1 t) + j\beta_2\sin(\omega_2 t) + j\phi_1(t) - j\phi_2(t)\right\}$$

$$=\sum_n\sum_l J_n(\beta_1)\ J_l(\beta_2)(-1)^n \exp\ (jn\omega_1 t + jl\omega_2 t + j\phi_1(t) - j\phi_2(t)) \tag{B.1.4}$$

where $\beta_1 = K_1 p_o A_1$ and $\beta_2 = K_1 p_o A_2$.

The following identities are known,

$$(A_1/2)\sin(\omega_1 t) = \frac{A_1}{4j}\Big[\exp(j\omega_1 t) - \exp(-j\omega_1 t)\Big]$$

$$(A_2/2)\sin(\omega_2 t) = \frac{A_2}{4j}\Big[\exp(j\omega_2 t) - \exp(-j\omega_2 t)\Big]$$

$$(A_1A_2/2)\cos((\omega_1-\omega_2)t) = \frac{A_1A_2}{8}\Big[\exp(j(\omega_1-\omega_2)t) + \exp(-j(\omega_1-\omega_2)t)\Big]$$

$$(A_1A_2/2)\cos((\omega_1+\omega_2)t) = \frac{A_1A_2}{8}\Big[\exp(j(\omega_1+\omega_2)t) + \exp(-j(\omega_1+\omega_2)t)\Big]$$

Substituting the above identities in eq. (B.1.1) and using the reseut of eq. (B.1.4) in eq. (B.1.1) we get the following expression

$$p_o^2 \sum_n \sum_l J_n(\beta_1)\, J_l(\beta_2)(-1)^n \exp\,(jn\omega_1 t + jl\omega_2 t + j\phi_1(t) - j\phi_2(t))$$

$$+ p_o^2 \sum_n \sum_l \frac{A_1}{4j} J_n(\beta_1)J_l(\beta_2)(-1)^n \exp(j\omega_1(n+1)t+jl\omega_2 t+j\phi_1(t)-j\phi_2(t))$$

$$+ p_o^2 \sum_n \sum_l \frac{-A_1}{4j} J_n(\beta_1)J_l(\beta_2)(-1)^n \exp(j\omega_1(n-1)t+jl\omega_2 t+j\phi_1(t)-j\phi_2(t))$$

$$+ p_o^2 \sum_n \sum_l \frac{A_2}{4j} J_n(\beta_1)J_l(\beta_2)(-1)^n \exp(j\omega_1 nt+j(l+1)\omega_2 t+j\phi_1(t)-j\phi_2(t))$$

$$+ p_o^2 \sum_n \sum_l \frac{-A_2}{4j} J_n(\beta_1)J_l(\beta_2)(-1)^n \exp(j\omega_1 nt+j(l-1)\omega_2 t+j\phi_1(t)-j\phi_2(t))$$

$$+ p_o^2 \sum_n \sum_l \frac{A_1 A_2}{8} J_n(\beta_1)J_l(\beta_2)(-1)^n \exp(j\omega_1(n+1)t+j(l-1)\omega_2 t +j\phi_1(t)-j\phi_2(t))$$

$$+ p_o^2 \sum_n \sum_l \frac{A_1 A_2}{8} J_n(\beta_1)J_l(\beta_2)(-1)^n \exp(j\omega_1(n-1)t+j(l+1)\omega_2 t +j\phi_1(t)-j\phi_2(t))$$

$$+ p_o^2 \sum_n \sum_l \frac{-A_1 A_2}{8} J_n(\beta_1)J_l(\beta_2)(-1)^n \exp(j\omega_1(n+1)t+j(l+1)\omega_2 t +j\phi_1(t)-j\phi_2(t))$$

$$+ p_o^2 \sum_n \sum_l \frac{-A_1 A_2}{8} J_n(\beta_1)J_l(\beta_2)(-1)^n \exp(j\omega_1(n-1)t+j(l-1)\omega_2 t +j\phi_1(t)-j\phi_2(t))$$

(B.1.5

Collecting the like frequency terms in the above expression we can simplify it to the following expression.

$$\mathrm{Re}\left\{p_o^2 \sum_n \sum_l (A_{nl} + jB_{nl}) \exp(jn\omega_1 t + jl\omega_2 t + j\phi_1 t - j\phi_2 t)\right. \quad (B.1.6)$$

where

$$A_{nl} = J_n(\beta_1)\, J_l(\beta_2)\, (-1)^n + \frac{A_1 A_2}{8} J_{n-1}(\beta_1)\, J_{l+1}(\beta_2)(-1)^{n-1}$$

$$+ \frac{A_1 A_2}{8} J_{n+1}(\beta_1)\, J_{l-1}(\beta_2)(-1)^{n+1} - \frac{A_1 A_2}{8} J_{n-1}(\beta_1)\, J_{l-1}(\beta_2)(-1)^{n\ 1}$$

$$- \frac{A_1 A_2}{8} J_{n+1}(\beta_1)\, J_{l+1}(\beta_2)(-1)^{n+1}$$

and

$$B_{nl} = -\frac{A_1}{4} J_{n-1}(\beta_1)\, J_l(\beta_2)(-1)^{n-1} - \frac{A_1}{4} J_{n+1}(\beta_1)\, J_l(\beta_2)(-1)^{n+2}$$

$$- \frac{A_2}{4} J_n(\beta_1)\, J_{l-1}(\beta_2)(-1)^n - \frac{A_2}{4} J_n(\beta_1)\, J_{l+1}(\beta_2)(-1)^{n+1}$$

Equation (B.1.6) can be compactly written as the following

$$\mathrm{Re}\left\{p_o^2 \sum_n \sum_l C_{nl} \exp(jn\omega_1 t + jl\omega_2 t + j\phi_1 t - j\phi_2 t)\right.$$

where $C_{nl} = A_{nl} + jB_{nl}$.

APPENDIX - B.2

To compute the Fourier transform of

$$1/2 \ \mathrm{Re} \left\{ \sum_n \sum_l C_{nl} \, C_{nl}^* \exp \left(jn\omega_1\tau + j\omega_2\tau \right) \exp \left(-|\tau| \ 2\pi(\Delta\nu_1 + \Delta\nu_2) \right) \right\} \tag{B.2.1}$$

We know the following results

a) $\mathrm{Re}(A_c) = \frac{1}{2} A_c + \frac{1}{2} A_c^*$

b) $\mathcal{F}_T \left[\exp(-|\tau| \ 2\pi(\Delta\nu_1 + \Delta\nu_2)) \right] = \left[\dfrac{2 \ (\pi\Delta\nu)^{-1}}{1 + \left[\dfrac{2(f - f_{nl})}{\Delta\nu} \right]^2} \right.$

where $\Delta\nu = \Delta\nu_1 + \Delta\nu_2$

c) If $G(f)$ is Fourier transform of $g(t)$ then

$$\mathcal{F}_T \left[\exp(j\omega_o t) \ g(t) \right] = G(\omega - \omega_o)$$

d) $\mathcal{F}_T \left[A(t) + g(t) \right] = \mathcal{F}_T \left[A(t) \right] + \mathcal{F}_T \left[g(t) \right]$

Applying the result given in (a) to eq. (B.2.1) we get

$$1/4 \left\{ \sum_n \sum_l C_{nl} \, C_{nl}^* \exp \left(jn\omega_1\tau + j\omega_2\tau \right) \exp \left(-|\tau| \ 2\pi(\Delta\nu_1 + \Delta\nu_2) \right) \right.$$
$$\left. + \text{ complex conjugate} \right\}$$

By applying the results given in (b), (c) and (d) in eq. (B.2.1) for computing the Fourier transform,

we get

$$S_X(f) = 1/4 \sum_n \sum_l C_{nl} C^*_{nl} \left\{ \frac{2}{(\pi\Delta\nu)} \left[\frac{1}{1 + \left[\frac{2(f - f_{nl})}{\Delta\nu} \right]^2} + \frac{1}{1 + \left[\frac{2(f + f_{nl})}{\Delta\nu} \right]^2} \right] \right\} \tag{3.17}$$

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